EuroGENESIS Workshop

STELLAR MODELING: Mixing, Convection, Rotation and Mass Loss

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Rotation: implementation and needs

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Why do we spend our time to study the effects of rotation on the evolution of the stars?

Why do we spend our time to study the **effects** of rotation on the evolution of the stars?

Well.....because stars....simply rotate

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doi:10.1088/0004-637X/722/1/605

A STELLAR ROTATION CENSUS OF B STARS: FROM ZAMS TO TAMS

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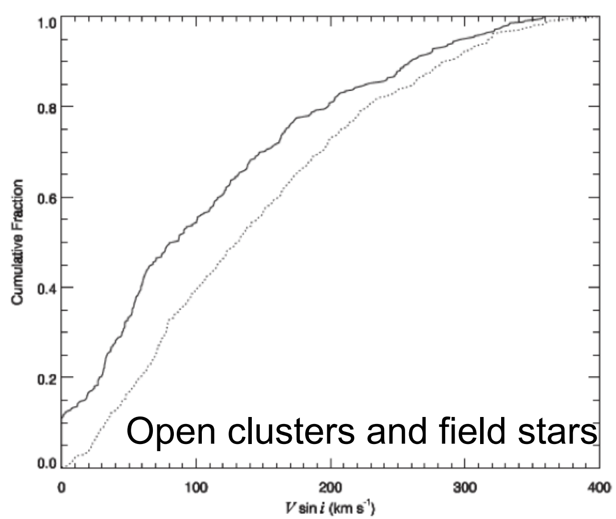

Figure 3. Cumulative distribution functions of projected rotational velocity for field (solid line) and cluster B stars (dotted line).

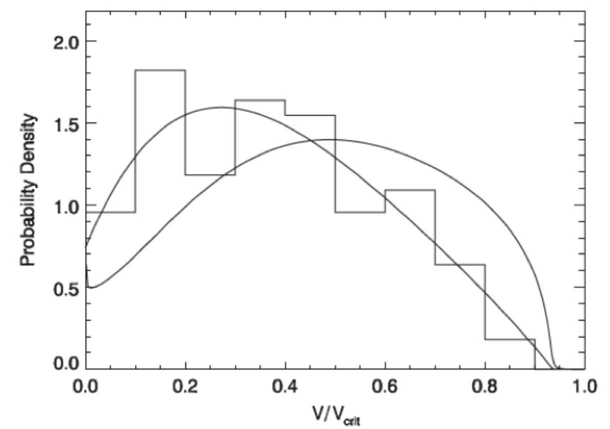

Figure 6. $V \sin i / V_{crit}$ histogram of all young B stars in our sample with $\log g_{polar} > 4.15$. Its polynomial fit is plotted as a thin solid line. The V_{eq}/V_{crit} distribution curve deconvolved from the polynomial fit is plotted as a thick solid line.

Hence we hope that its inclusion will help us to better understand them and the world out there

A&A 496, 841–853 (2009)
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Astronomy & Astrophysics

The VLT-FLAMES survey of massive stars: constraints on stellar evolution from the chemical compositions of rapidly rotating Galactic and Magellanic Cloud B-type stars⋆,⋆⋆

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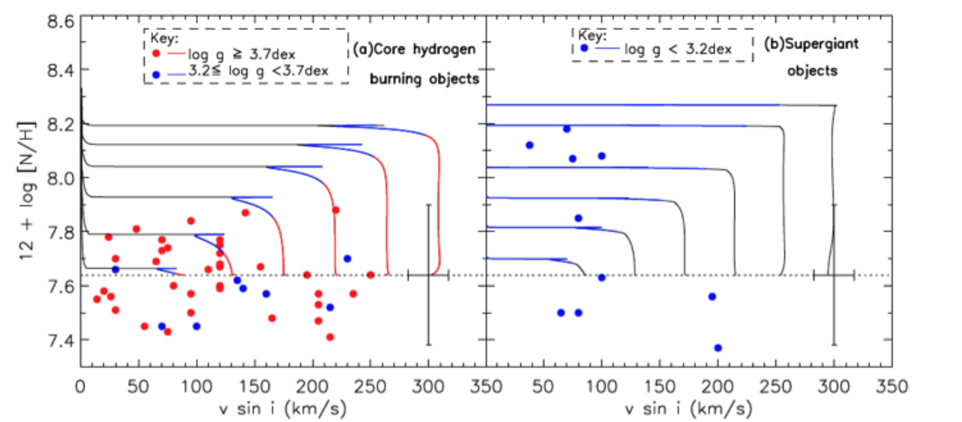
Received 7 April 2008 / Accepted 5 January 2009

ABSTRACT

Aims. We have previously analysed the spectra of 135 early B-type stars in the Large Magellanic Cloud (LMC) and found several groups of stars that have chemical compositions that conflict with the theory of rotational mixing. Here we extend this study to Galactic and Small Magellanic Cloud (SMC) metallicities.
Methods. We provide chemical compositions for ~50 Galactic and ~100 SMC early B-type stars and compare these to the LMC results. These samples cover a range of projected rotational velocities up to ~300 km s^{-1} and hence are well suited to testing rotational

Chemical compositions are presented for 53 Galactic and 96 SMC stars and compared with the results for the 135 LMC stars from Paper VII. In order to investigate the role of rotational mixing, a large population of fast rotators is necessary. Our targets have projected rotational velocities up to ~300 km s^{-1} and hence



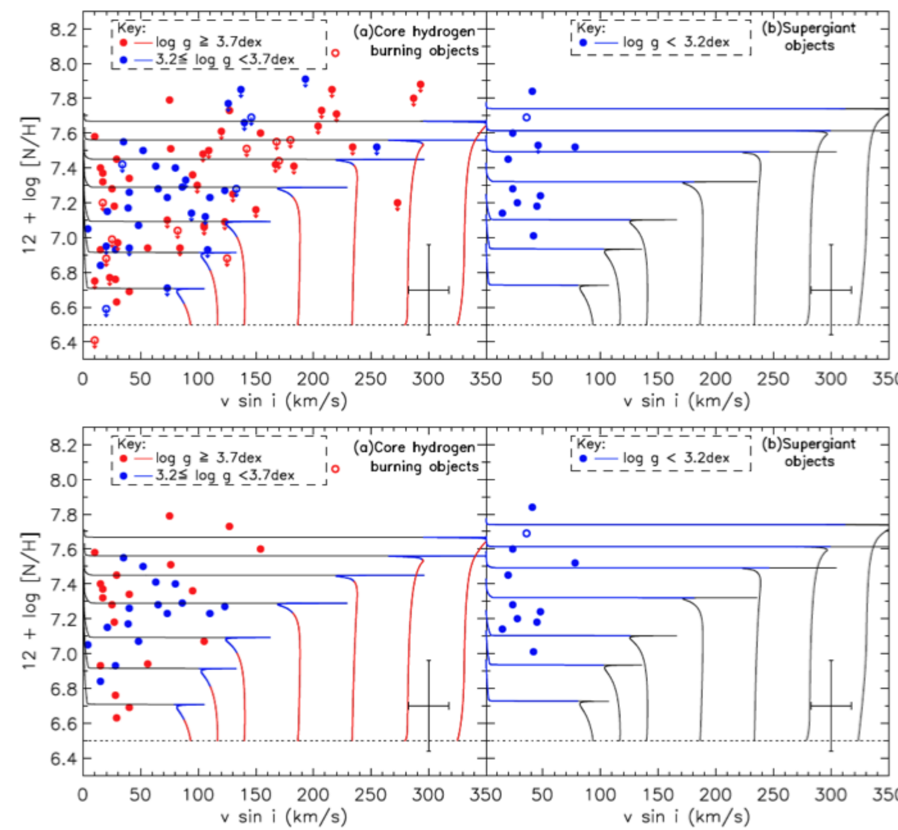


Fig. 7. Nitrogen abundance (12 + log[N/H]) as a function of projected rotational velocity for the SMC sample of stars. Symbols are equivalent to those in Fig. 5. The *lower panel* is equivalent to the *upper panel* except that upper limits to the nitrogen abundances have been removed. Evolutionary models are plotted for 12 and 13 $M_\odot$ in panels **a)** and **b)**.

Rotation basics

To keep a long story short...
...it was recognized long time ago that it is possible to simulate the influence of rotation on the structural shape of a star with a 1D code by adopting three reasonable assumptions:

1) **Shellular rotation**

Since no work must be done to move on an isobar, Zahn (1992) proposed that both the chemical composition and the angolar velocity are constant on an isobar as a consequence of a “vigorous” horizontal mixing.

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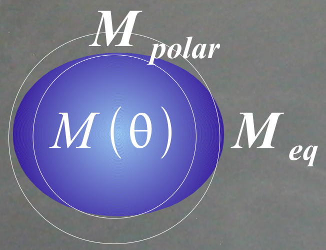
2) <u>Roche approximation</u>



Mass strongly centrally concentrated

$$M(\theta) = M_{const}$$

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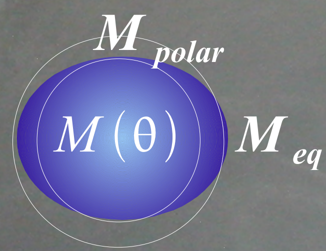

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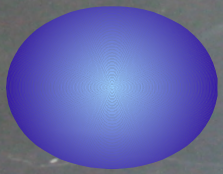
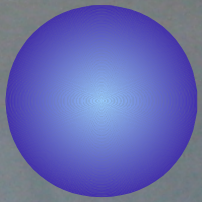
3) Equivalent volumes



$$V_\Psi = \int_r \int_\Psi dn\, d\sigma$$

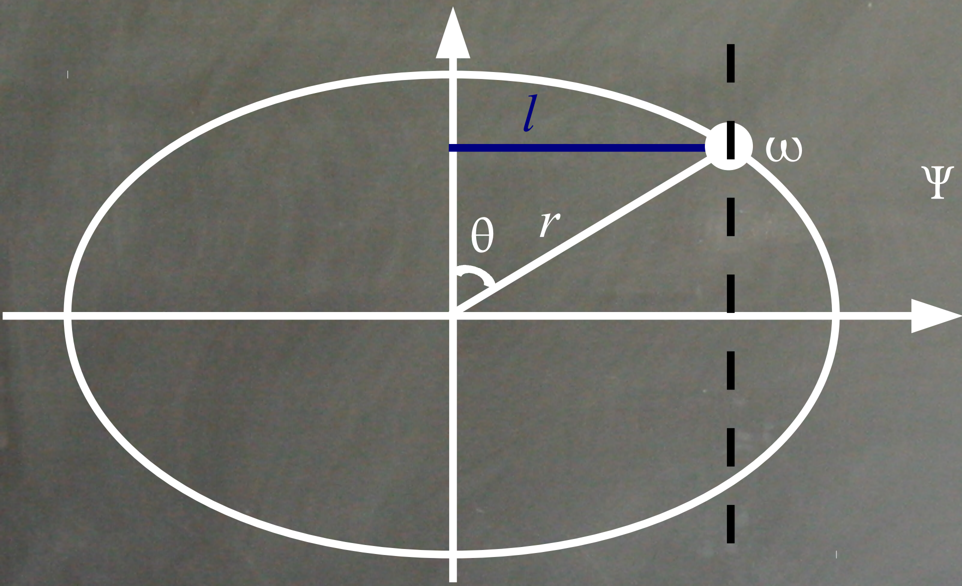


$$V_\Psi = \frac{4}{3}\pi r_\Psi^3$$

Two different cases should, in principle, be considered:

Conservative case

if ω has cylindrical symmetry it is possible to define a total potential Ψ:



$$\Psi = -\frac{GM_\Psi}{r} - \frac{1}{2}\omega^2 l^2 = -\frac{GM_\Psi}{r} - \frac{1}{2}\omega^2 r^2 \sin^2\theta$$

$$\frac{dP}{dM_\Psi} = -\frac{G\,M_\Psi}{4\pi\, r_\Psi^4} \cdot f_P$$

$$\frac{dM}{dr_\Psi} = 4\pi\, r_\Psi^2\, \rho$$

$$\frac{d\,lnT_\Psi}{d\,lnP_\Psi} = \frac{3\kappa_\Psi L_\Psi P_\Psi}{16\pi\, a\, c\, G\, T_\Psi^4 M_\Psi} \cdot \frac{f_T}{f_p}$$

$$dL = \epsilon_\Psi\, \Delta M$$

$$f_P = \frac{4\pi\, r_\Psi^4}{G\, M_\Psi S_\Psi \langle g_{eff}^{-1} \rangle}$$

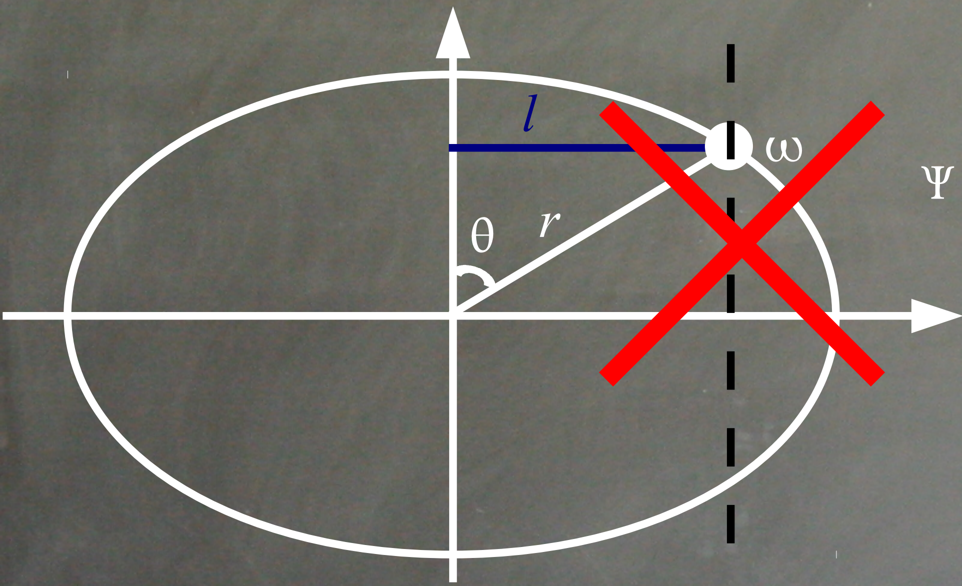
$$f_T = \frac{16\pi^2 r_\Psi^4}{S_\Psi^2 \langle g_{eff}^{-1} \rangle \langle g_{eff} \rangle}$$

$$\langle g_{eff} \rangle = \frac{1}{S_\Psi} \int_\Psi g_{eff}\, d\sigma$$

$$S_\Psi = \int_\Psi d\sigma = \int_\Psi r^2 \sin\theta\, d\theta\, d\phi$$

NON Conservative case

The same surface Ψ is not any more the potential but it is still an isobar:



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$$f_P = \frac{4\pi r_\Psi^4}{G M_\Psi S_\Psi <g_{eff}^{-1}>}$$

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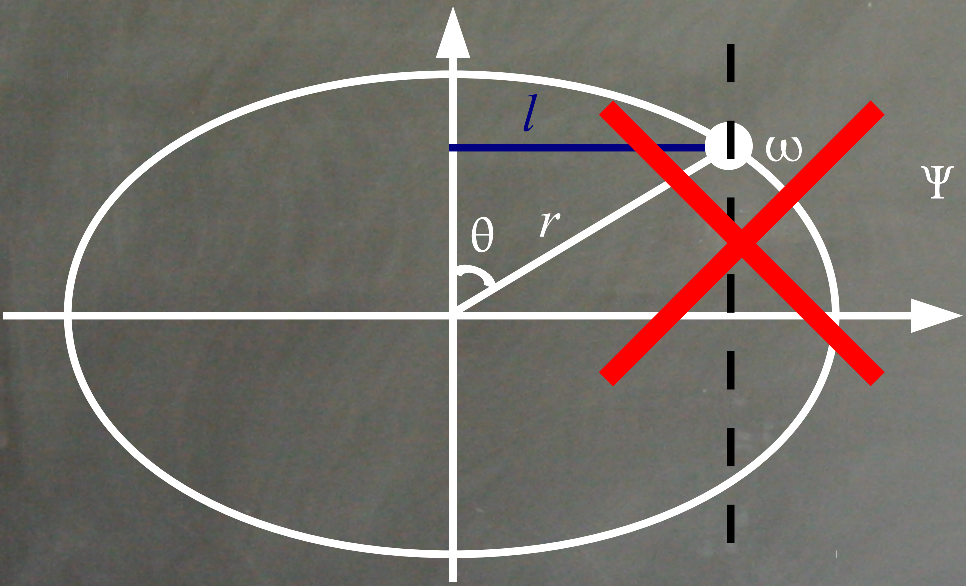

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$$\frac{d\ln \overline{T}_\Psi}{d\,lnP_\Psi} = \frac{3\,\overline{\kappa}_\Psi L_\Psi P_\Psi}{16\pi\, a\, c\, G\, \overline{T}_\Psi^4 M_\Psi}\cdot\frac{f_T}{f_p}$$

$$dL = \overline{\epsilon}_\Psi \Delta M$$

$$f_P = \frac{4\pi\, r_\Psi^4}{G\,M_\Psi S_\Psi < g_{eff}^{-1} >}$$

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$$S_\Psi = \int_\Psi d\sigma = \int_\Psi r^2 \sin\theta\, d\theta\, d\phi$$

Rotation: basics

Common assumptions:

Shellular rotation	⇒	ω and c.c. constant on an isobar
Roche approximation	⇒	mass centrally concentrated
Equivalent volumes	⇒	Adoption of the radii of the equivalent spheres

Angular velocity ω:

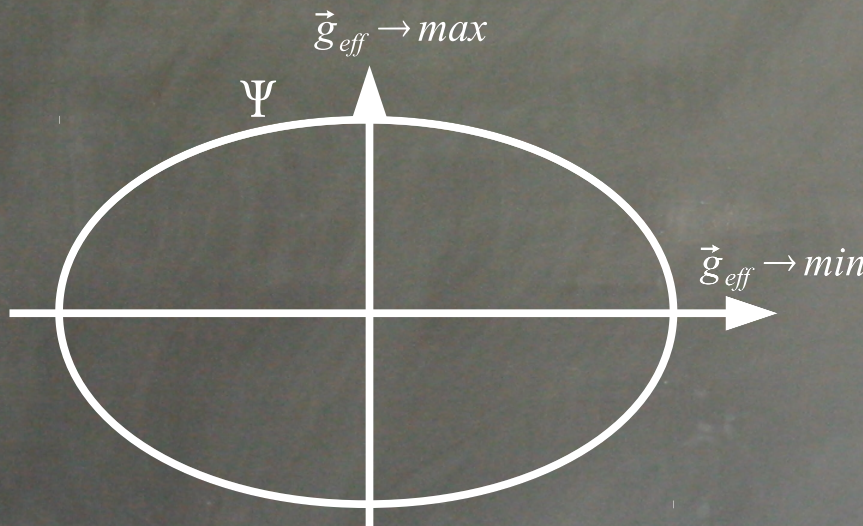
	cylindrical symmetry (admits a potential)	no restrictions (no potential exists)
Advantages	ρ and T constant on an isobar (also κ and ε)	Shellular rotation only
Disadvantages	Solid body rotation	ρ and T vary on an isobar (also κ and ε)

… but in practice there is no difference (in a 1D code) between the cylindrical (conservative) and NON cylindrical (non conservative) cases, it is just a different interpretation of the physical quantities: constant versus average values on an isobar...

…but the next step is really the crucial one when talking of rotation: the treatment of the instabilities that may lead to the transport of the angular momentum and the mixing of the chemical composition ...

Question: which are the main instabilities in a rotating star?

Rotational instabilities: meridional circulation



The first one to notice that these two equations cannot be simultaneously fulfilled in radiative equilibrium was Von Zeipel (1924)

$$\vec{F}_\Psi(r,\theta,\phi) = f(\Psi)\,\vec{g}_{eff}(r,\theta,\phi)$$

$$\vec{\nabla}\cdot\vec{F}_\Psi(r,\theta,\phi) = \epsilon_\Psi$$

CIRCOLAZIONE INTERNA E INSTABILITÀ NELLE BINARIE STRETTE 17

l' equatore verso i poli, mentre a profondità maggiore la corrente si dirigerà dai poli all' equatore.

Procedendo verso l' interno, al crescere di ρ si raggiungerà una superficie equipotenziale in cui la densità ha il valore ρ^* dato dalla relazione:

$$\rho^* = \frac{\omega^2}{2\pi G}.$$

Questa superficie è caratterizzata dal fatto che sopra di essa la componente verticale della velocità è zero; essa non viene attraversata da cor-

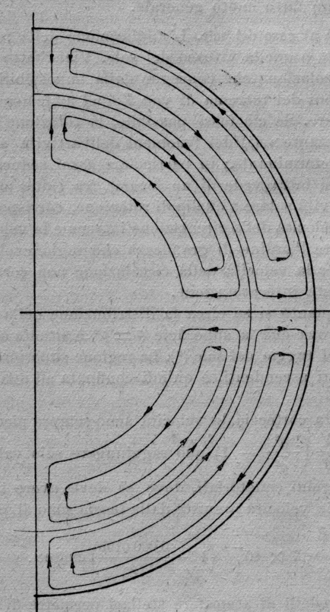
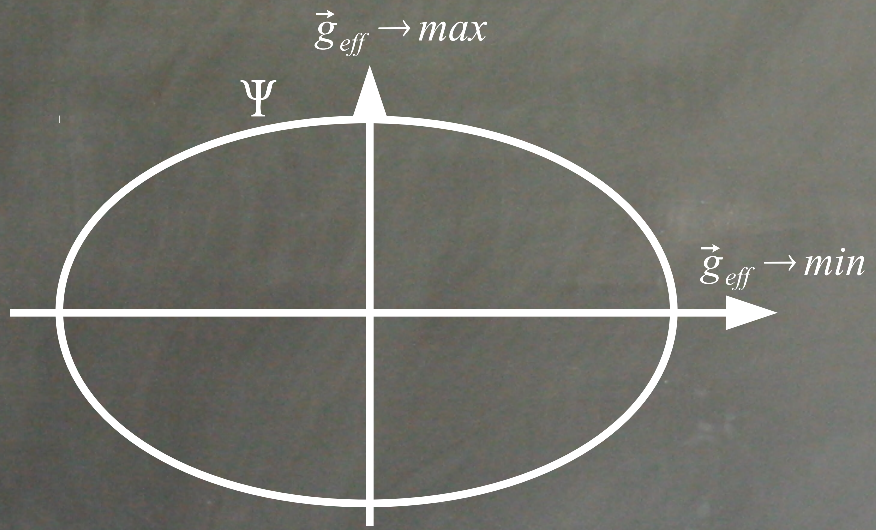


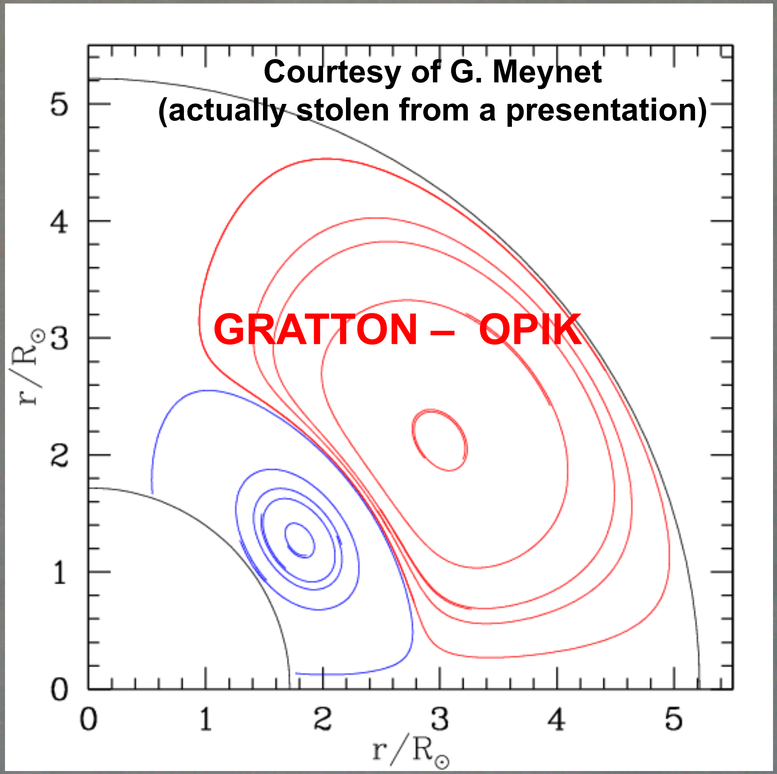
Fig. 1

renti convettive e quindi separa la stella in due regioni distinte, fra le quali la circolazione corrispondente alla teoria qui svolta non prevede scambi di materia. In pratica è difficile pensare ad una separazione così

GRATTON – April 1944

Rotational instabilities: meridional circulation





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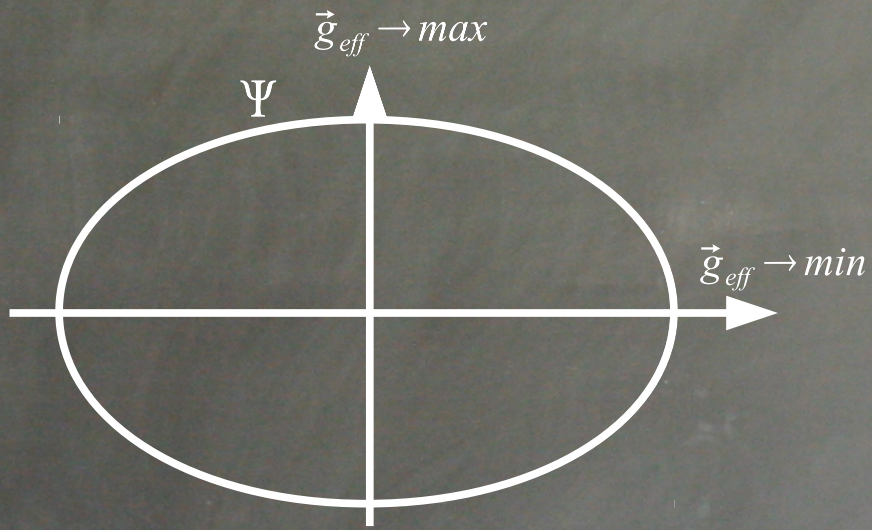
Rotational instabilities: meridional circulation



$$\vec{\nabla}\cdot\vec{F}_{rad}=\rho\,\epsilon_{nuc}-\vec{\boldsymbol{u}}\cdot(c_P\,\rho\,\vec{\nabla}\,T-\delta\,\vec{\nabla}\,P)-c_P\,\rho\frac{\partial T}{\partial t}+\delta\frac{\partial P}{\partial t}$$

Kippenhahn (1974)

$$\boldsymbol{u}_{ES}=\left(\frac{L}{M}\,\frac{\omega^2}{4\pi G\rho g\delta}\right)\left(\frac{\nabla_{ad}}{\nabla_{ad}-\nabla_{rad}}\right)$$

Kippenhahn & Mollenhoff (1974)

$$\boldsymbol{u}_{ES}=\left[\frac{\rho\,\epsilon}{g}-\frac{L}{Mg}\left(1+\frac{\omega^2}{2\pi G}\right)\right]\left(\frac{1}{\delta\,\rho}\right)\left(\frac{\nabla_{ad}}{\nabla_{ad}-\nabla}\right)$$

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$$\vec{F}_{\Psi}(r,\theta,\phi)=f(\Psi)\,\vec{g}_{eff}(r,\theta,\phi)$$

$$\vec{\nabla}\cdot\vec{F}_{\Psi}(r,\theta,\phi)=\epsilon_{\Psi}$$

Kippenhahn & Weigert (1990)

$$\boldsymbol{u}_{ES}=\frac{8}{3}\,\frac{\omega^2 r}{g}\,\frac{L}{Mg}\,\frac{\gamma-1}{\gamma}\,\frac{1}{\nabla_{ad}-\nabla}\left(1-\frac{\omega^2}{2\pi G\rho}\right)$$

Maeder & Zahn (1998)

Nightmare

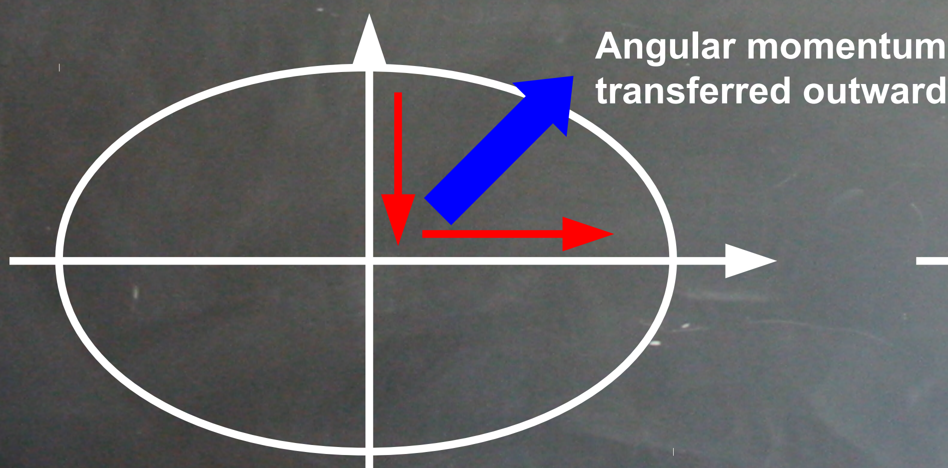
Rotational instabilities: meridional circulation

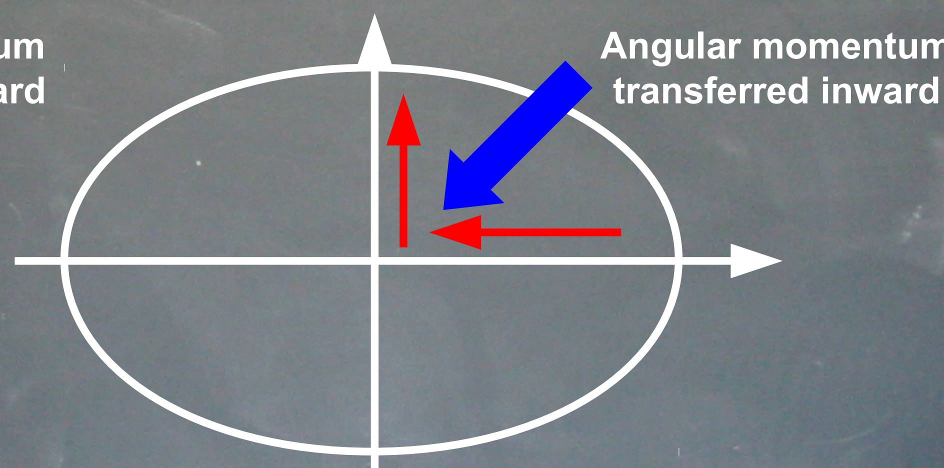
Expression of the meridional circulation as provided by Maeder & Zahn (1998)

$$U=\frac{P}{\rho\,\bar{g}\,C_P\,T\,[\nabla_{ad}-\nabla+(\varphi/\delta)\nabla_\mu]}\left\{\frac{L}{M_x}\left[\frac{8}{3}\frac{\omega^2 r^3}{GM}\left(1-\frac{\omega^2}{2\pi G\rho}-\frac{\bar{\varepsilon}+\varepsilon_g}{\varepsilon_m}\right)-\frac{\rho_m}{\rho}\left(\frac{r}{3}\frac{d}{dr}A-2\frac{H_T}{r}\left(1+\frac{D_h}{K}\right)\frac{\Theta}{\delta}+\frac{2}{3}\Theta\right)-\frac{\bar{\varepsilon}+\varepsilon_g}{\varepsilon_m}\left(A+f_\varepsilon\,\varepsilon_T\frac{\Theta}{\delta}+(1-f_\varepsilon)\Theta\right)-\frac{\omega^2}{2\pi G\rho}\Theta\right]+\frac{C_p T}{\delta}\frac{\partial\Theta}{\partial t}\right\}$$

$$\Theta\equiv\frac{\tilde{\rho}}{\bar{\rho}}=\frac{1}{3}\frac{r^2}{g}\frac{d\,\omega^2}{dr} \qquad A=H_T\frac{d}{dr}\left(\frac{\Theta}{\delta}\right)-(\chi_T+1-\delta)\frac{\Theta}{\delta}$$

In principle meridional circulation moves matter through the star and hence it can both transport angular momentum and induce mixing of the chemical composition

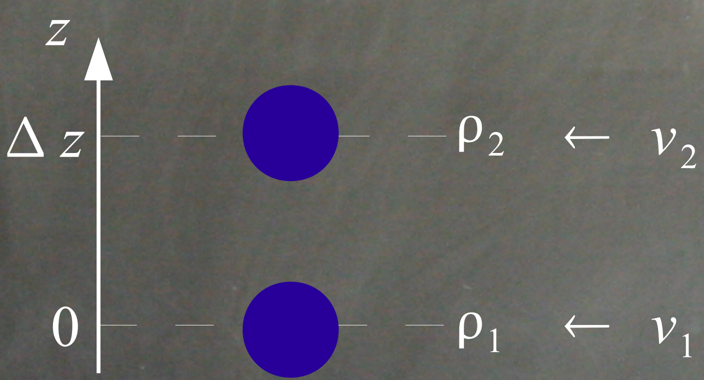




Rotational instabilities

Are there additional instabilities induced by rotation?

Dynamical Shear



Restoring force $$f = -\frac{\partial \rho}{\partial z} \Delta z \cdot g \cdot \Delta V$$

Energy $$E_{restoring} = f \cdot \Delta z$$

If the star rotates differentially, the extra energy of an eddy brought from layer 1 to layer 2 is given by:

$$E_{turbulent} = \Delta M (\Delta v)^2 = \rho \Delta V \left(\frac{\partial v}{\partial z} \Delta z \right)^2$$

$$R = \frac{E_{restoring}}{E_{turbulent}} = -\frac{\frac{g}{\rho}\frac{\partial \rho}{\partial z}}{(\partial v / \partial z)^2} = \frac{N^2}{(\partial v / \partial z)^2}$$

$$R = \frac{E_{restoring}}{E_{turbulent}} = \left\{ \frac{g^2}{P} \delta \left[\nabla_{ad} - \nabla + \frac{\varphi}{\delta} \left(\frac{\partial \ln \mu}{\partial \ln P} \right) \right] \right\} / \left[\left(\frac{\partial \omega}{\partial \ln r} \right)^2 \right] = \frac{N^2 \, (\partial \ln r / \partial \omega)^2}{\rho} = (N_T^2 + N_\mu^2)(\partial \ln r / \partial \omega)^2 < \frac{1}{4}$$

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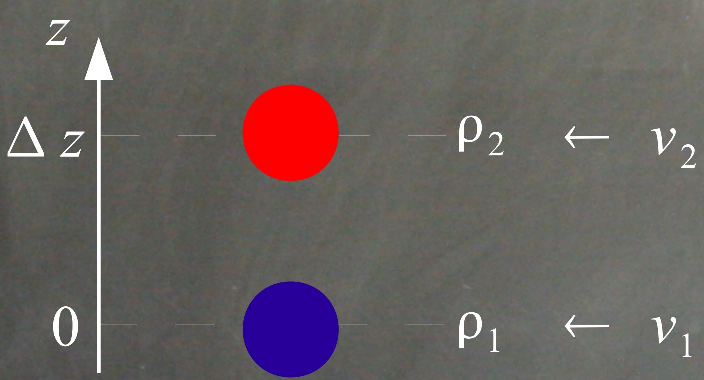


But we are clever…
thermal losses reduce the restoring force…
as well as the Horizontal currents...

$$R = \frac{E_{restoring}}{E_{turbulent}} = \left(N_T^2 + N_\mu^2\right)\frac{\left(\partial \ln r / \partial \omega\right)^2}{\rho} = \left(\frac{\Gamma_T}{\Gamma_T+1} N_T^2 + \frac{\Gamma_\mu}{\Gamma_\mu+1} N_\mu^2\right)\left(\partial \ln r / \partial \omega\right)^2 \quad < \frac{1}{4}$$

$$\Gamma_T = \frac{vl}{6\left(K + D_h\right)} \qquad \Gamma_\mu = \frac{vl}{6D_h}$$

Turbulent horizontal diffusivity (D_h)

If one also assumes that the eddies have a continuum spectrum of velocities v, also the idea of a strict criterion vanishes!

In other words there will be always some eddies for which R<1/4, so that any layer is in principle unstable with respect to the shear

We magically turn a strict on/off criterion

$$R=\left(\frac{\Gamma_T}{\Gamma_T+1}N_T^2+\frac{\Gamma_\mu}{\Gamma_\mu+1}N_\mu^2\right)\left(\partial\ln r/\partial\omega\right)^2$$

$$R=\left(\Gamma_T N_T^2+\Gamma_\mu N_\mu^2\right)\left(\partial\ln r/\partial\omega\right)^2$$

in

$$R=\left(\frac{vl}{6\left(k+D_h\right)}N_T^2+\frac{vl}{6\mathrm{D}_h}N_\mu^2\right)\left(\partial\ln r/\partial\omega\right)^2$$

$$R=\frac{vl}{3}\left(\frac{N_T^2}{2\left(k+D_h\right)}+\frac{N_\mu^2}{2\mathrm{D}_h}\right)\left(\partial\ln r/\partial\omega\right)^2$$

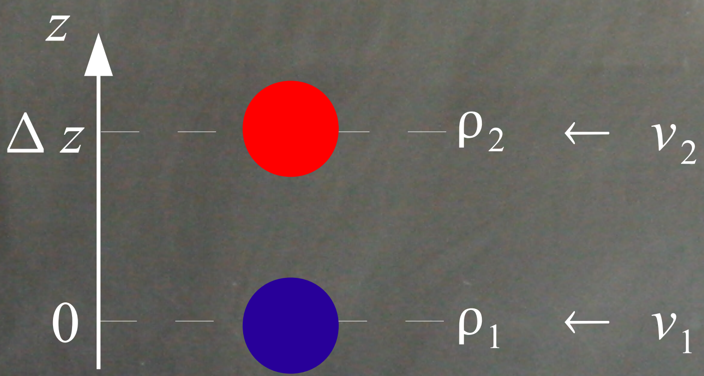
a diffusion coefficient always at work

$$D_{shear}=\frac{vl}{3}=2\,\frac{R\left(\partial\omega/\partial\ln r\right)^2}{N_T^2/\left(K+D_h\right)+N_\mu^2/D_h}$$

Rotational instabilities

Are there additional instabilities induced by rotation?

… just ... Shear



But we are clever…
thermal losses reduce the restoring force…
as well as the Horizontal currents...

$$R = \frac{E_{restoring}}{E_{turbulent}} = (N_T^2 + N_\mu^2)\frac{(\partial \ln r/\partial \omega)^2}{\rho} = \left(\frac{\Gamma_T}{\Gamma_T+1}N_T^2 + \frac{\Gamma_\mu}{\Gamma_\mu+1}N_\mu^2\right)(\partial \ln r/\partial \omega)^2 < \frac{1}{4}$$

$$\Gamma_T = \frac{vl}{6(K+D_h)} \quad \Gamma_\mu = \frac{vl}{6D_h}$$

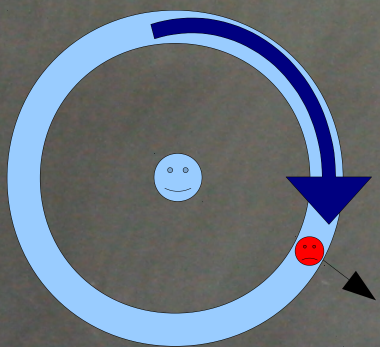
Turbulent horizontal diffusivity (D_h)

$$D_{shear} = \frac{8}{5}\frac{1/4(r\,d\omega/dr)^2}{N_T^2/(K+D_h) + N_\mu^2/D_h} \qquad N_T^2 = \frac{g\delta}{H_P}(\nabla_{ad} - \nabla_{rad}) \qquad N_\mu^2 = \frac{g\delta}{H_P}\left(\frac{\phi}{\delta}\nabla_\mu\right)$$

Maeder & Zahn (1998) – Meynet and Maeder (2003)

Rotational instabilities

What about the turbulent horizontal diffusivity D_h?



Chaboyer & Zahn 1992 AA 253,173

In fact, the meridional velocity and the horizontal diffusivity are strongly correlated: the horizontal turbulence obviously vanishes if there is no circulation. At the present, primitive stage of the theory, it does not seem unreasonable to assume that D_H and U_2 are proportional, and thus to state that

$$\frac{|rU_2|}{D_H} = C_H, \tag{23}$$

C_H being a parameter of order unity. A more refined prescription

Zahn 1992 AA 265,115

As we have seen, two transport coefficients remain, which cannot be derived from first principles, namely the horizontal component of the turbulent viscosity ν_h, and its companion, the horizontal diffusivity D_h. If we wish to proceed, we must content with some parametrization, whose arbitrariness can fortunately be limited by the few constraints that we have encountered.

Referring back to (2.11*b*), we note that the amplitude of the differential rotation will remain small only as long as ν_h is of the order of $|2V - \alpha U|$, or larger. The simplest way to implement this is to take

$$D_h = \frac{1}{C_h} r\, |2V - \alpha U|\,,$$

Maeder 2003 AA 399,263

Ω_2 between the two Eqs. (17) and (18). This gives for the coefficient of viscosity due to the horizontal turbulence

$$\nu_h = A\, r \left(r\overline{\Omega}(r)\, V\, [2V - \alpha U]\right)^{\frac{1}{3}}$$

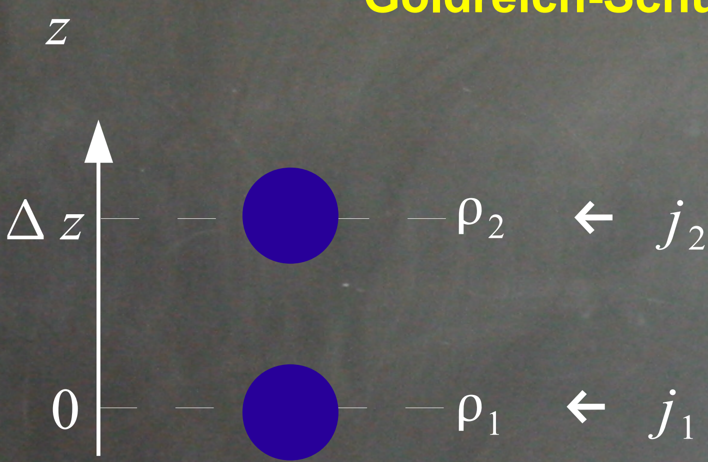
$$\text{with} \quad A = \left(\frac{3}{400 n\pi}\right)^{\frac{1}{3}}. \tag{19}$$

For $n = 1$, 3 or 5 $A \approx 0.134, 0.0927, 0.0782$ respectively. This

Rotational instabilities

Are there additional instabilities induced by rotation?

Let me just mention the
Solberg-Hoiland dynamical instability
and the
Goldreich-Schubert-Fricke (GSF) secular instability



$$\frac{dP}{dr} = -g\rho + \frac{\rho}{r^3} r^4 \Omega^2 = -g\rho + \frac{\rho}{r^3} j^2$$

Eddies move preserving their angular momentum j

$$\frac{\partial^2 r}{dt^2} = \left\{ -\frac{g}{\rho}\left[\left(\frac{\partial\rho}{\partial r}\right)_{eddy} - \left(\frac{\partial\rho}{\partial r}\right)_{env}\right] + \frac{\rho}{r^3}\left[\left(\frac{\partial j^2}{\partial r}\right)_{eddy} - \left(\frac{\partial j^2}{\partial r}\right)_{env}\right]\right\} r = \left(N_\rho^2 + N_j^2\right) r$$

Intrinsically negative

The SH instability grows only if j *<u>decreases</u>* outward

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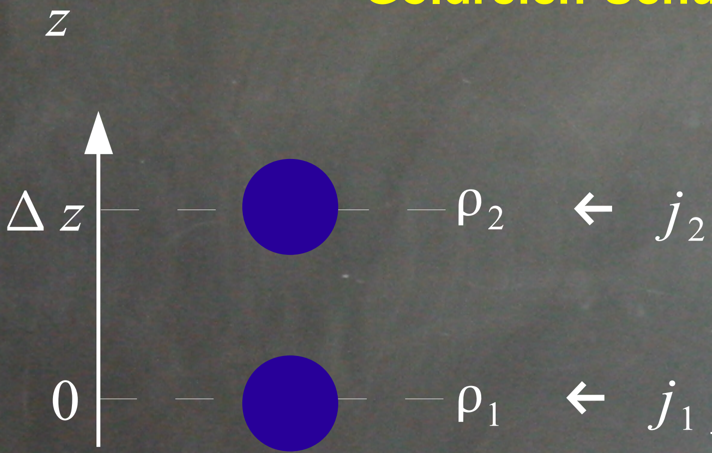

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Intrinsically negative

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Rotational instabilities: the transport of the angular momentum

$$\rho \frac{d}{dt}(r^2\omega)_{M_r} = \frac{1}{5}\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^4 \omega U) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 D_{shear}\frac{\partial \omega}{\partial r}\right)$$

This is an advective – diffusive equation

In order to find a stable solution for this equation (plus the nightmare expression for U), it is necessary to solve a system of four equations!

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<u>ALTERNATIVELY:</u>

the transport of the angular momentum is often computed by adopting a pure diffusive equation (e.g. Heger, Langer & Woosley 2000)

$$\rho \frac{d}{dt}(r^2 \omega)_{M_r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 (D_{shear} + D_{mc})\frac{\partial \omega}{\partial r}\right)$$

FRANEC 6.0

Major improvements compared to the release 4.0 (Limongi & Chieffi 2003, Chieffi & Limongi 2004) and 5.0 (Limongi & Chieffi 2006)

- FULL COUPLING of: Physical Structure - Nuclear Burning -
  + Chemical Mixing (convection, semiconvection, rotation)

- INCLUSION OF ROTATION: Transport of Angular Momentum (Advection/Diffusion)

- MASS LOSS (Enhanced mass loss for RSG phase, Van Loon 2005)

- TWO NUCLEAR NETWORKS H → Pb :
  197 isotopes (490 reactions) H/He Burning
  324 isotopes (3019 reactions) Advanced Burning

- SOLAR COMPOSITION (Asplund et al. 2009)

We have implemented both schemes: the advection+diffusion & the pure diffusive

FRANEC 6: current release 6.130329

$$\frac{dP}{dM_\Psi} = -\frac{G M_\Psi}{4\pi r_\Psi^4} \cdot f_P \qquad f_P = \frac{4\pi r_\Psi^4}{G M_\Psi S_\Psi <g_{eff}^{-1}>}$$

$$\frac{dM}{dr_\Psi} = 4\pi r_\Psi^2 \rho \qquad f_T = \frac{16\pi^2 r_\Psi^4}{S_\Psi^2 <g_{eff}^{-1}> <g_{eff}>}$$

$$\frac{d\, lnT_\Psi}{d\, lnP_\Psi} = \frac{3\kappa_\Psi L_\Psi P_\Psi}{16\pi a c G T_\Psi^4 M_\Psi} \cdot \frac{f_T}{f_p}$$

$$dL = \epsilon_\Psi \Delta M$$

$$\rho \frac{d}{dt}(r^2 \omega)_{M_r} = 0$$

$$\frac{dY_i}{dt} = \left(\frac{\partial Y_i}{\partial t_{nuc}}\right) + \frac{\partial}{\partial m}\left[(4\pi\rho r^2)^2 (D_{semi} + D_{mix} + D_{rot}) \frac{\partial x_i}{\partial m}\right] \qquad i = 1 \ldots N$$

$$1\ system\ of\ M_{meshes} \cdot (N_{isotopes} + 5)\ ODEs$$

$$\rho \frac{d}{dt}(r^2 \omega)_{M_r} = \frac{1}{5}\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^4 \omega U) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^4 D_{shear} \frac{\partial \omega}{\partial r}\right)$$

U(Maeder & Zahn 1998)

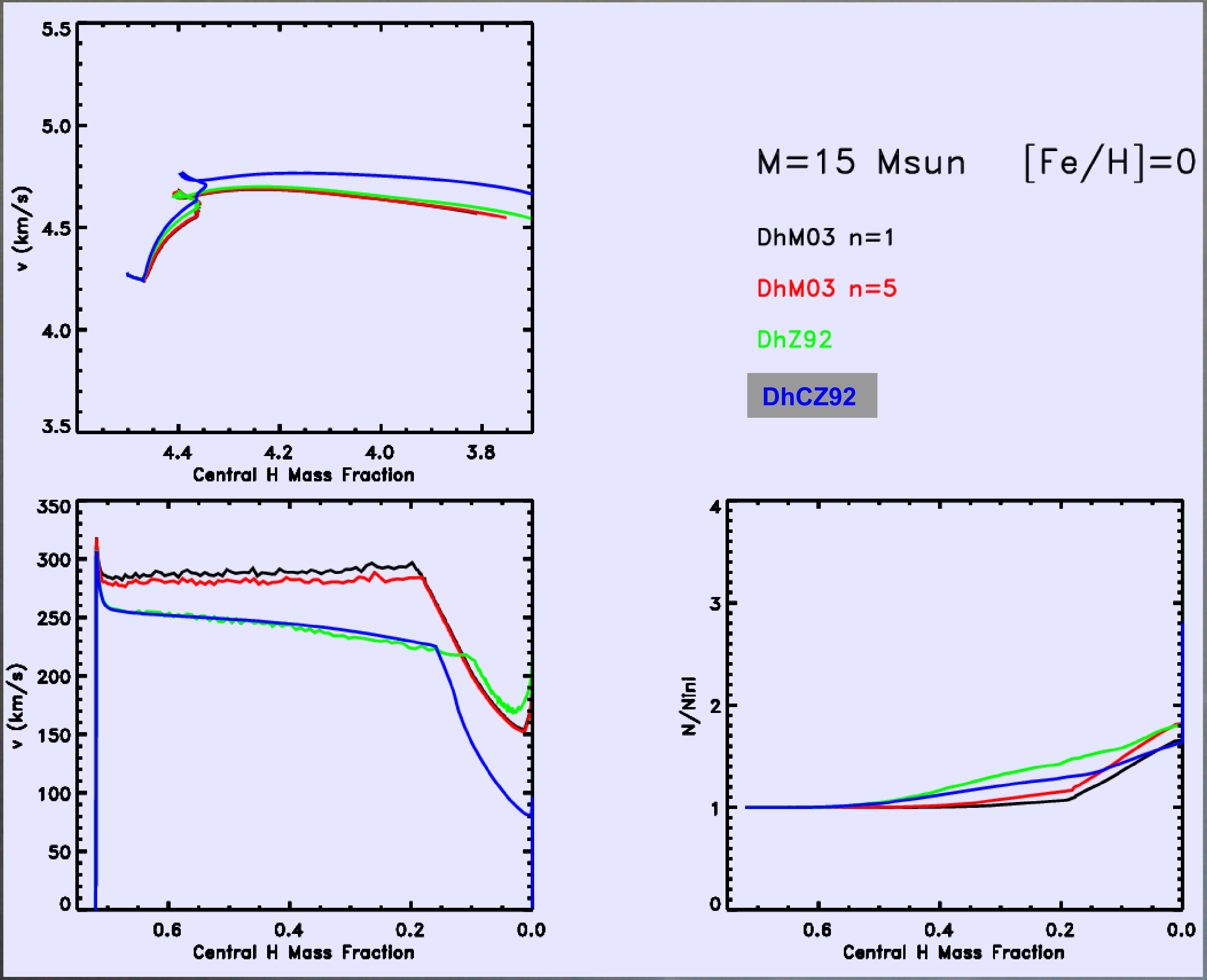
$\Leftarrow$ $4\ ODEs$

or

$$\rho \frac{d}{dt}(r^2 \omega)_{M_r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^4 \left(D_{shear} + \frac{1}{30} r |U|\right) \frac{\partial \omega}{\partial r}\right]$$

U(Maeder & Zahn simplified 1998)

Which turbulent horizontal diffusivity D_h use in the code?



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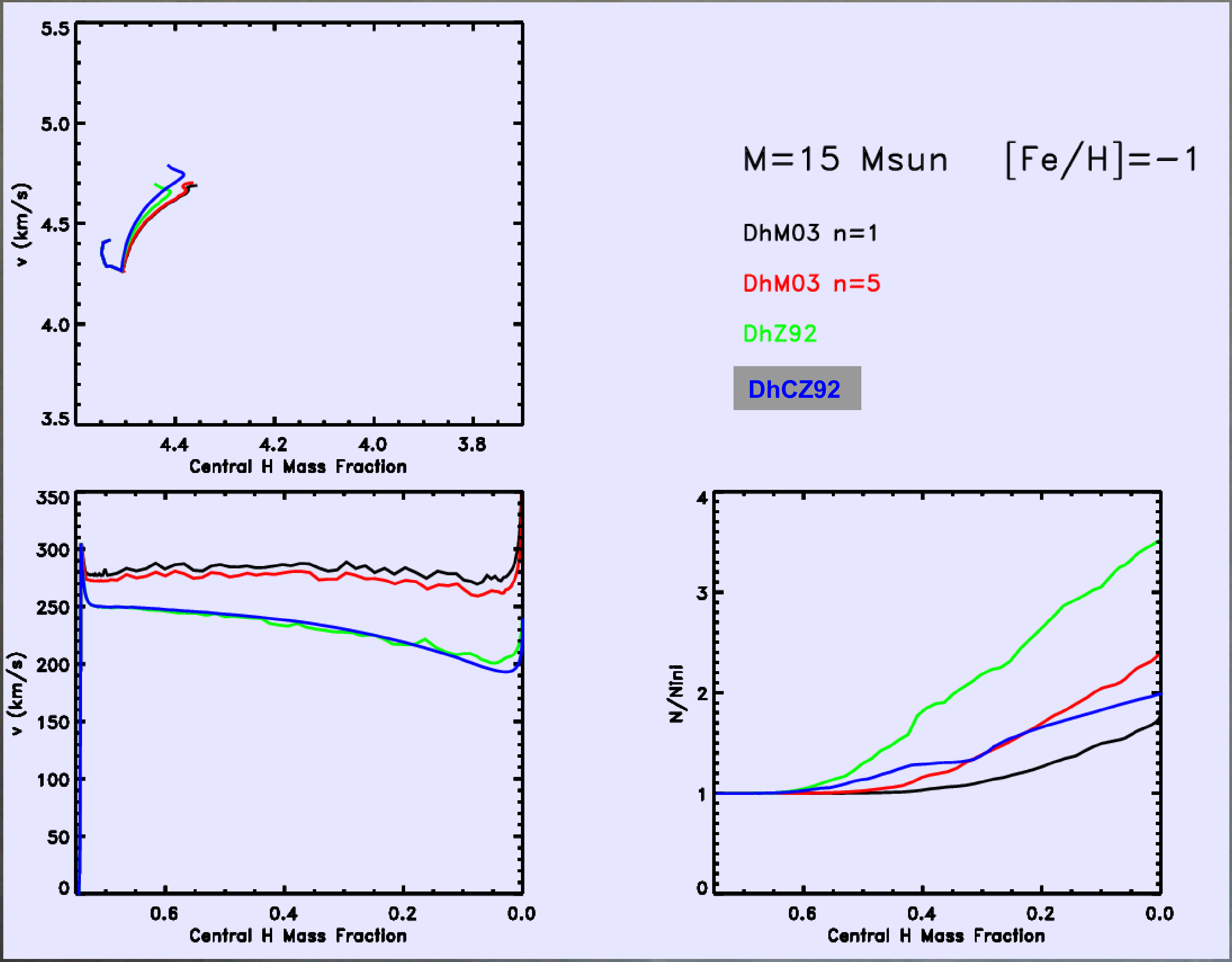

It is clear that some calibration is necessary!

We consider two free parameter directly connected to rotation:

f_c that multiplies the total diffusion coefficient D that controls the mixing due to the shear and the meridional circulation

f_μ that multiplies the gradient fo molecular weight

FRANEC 6: current release 6.130329

$$\frac{dP}{dM_\Psi} = -\frac{G M_\Psi}{4\pi r_\Psi^4} \cdot f_P \qquad f_P = \frac{4\pi r_\Psi^4}{G M_\Psi S_\Psi < g_{eff}^{-1} >}$$

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U(Maeder & Zahn 1998)

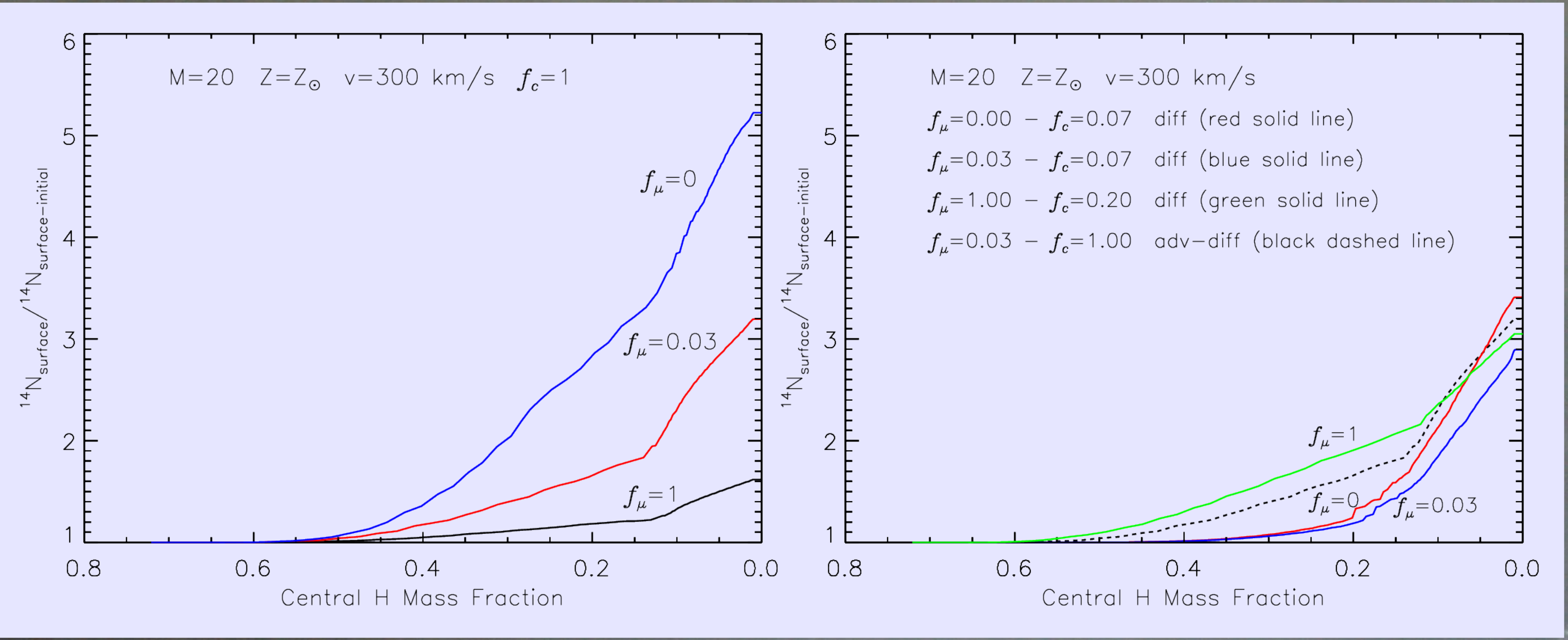
$\Leftarrow$ 4 $ODEs$

or

$$\rho \frac{d}{dt}(r^2 \omega)_{M_r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^4 \left(f_C D_{shear} + \frac{1}{30} r |U|\right) \frac{\partial \omega}{\partial r}\right]$$

U(Maeder & Zahn 1998 - simplified)

$$\nabla_\mu \rightarrow f_\mu \nabla_\mu$$

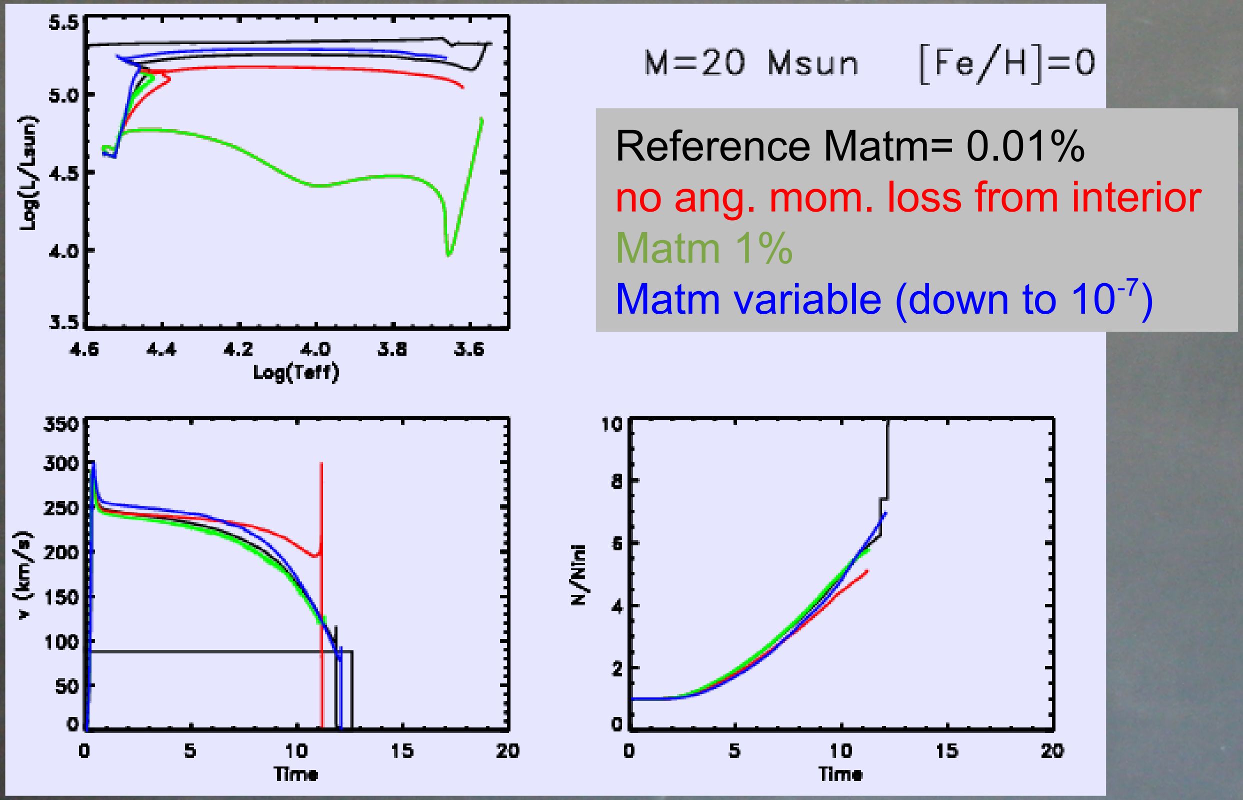


M=20 Z=Z⊙ v=300 km/s f_c=1
f_μ=0
f_μ=0.03
f_μ=1
$^{14}N_{surface}/^{14}N_{surface-initial}$
Central H Mass Fraction
M=20 Z=Z⊙ v=300 km/s
f_μ=0.00 – f_c=0.07 diff (red solid line)
f_μ=0.03 – f_c=0.07 diff (blue solid line)
f_μ=1.00 – f_c=0.20 diff (green solid line)
f_μ=0.03 – f_c=1.00 adv–diff (black dashed line)
f_μ=1
f_μ=0
f_μ=0.03
$^{14}N_{surface}/^{14}N_{surface-initial}$
Central H Mass Fraction

Just a couple of additional *technical* problems...

Where do you extract the angular momentum from?

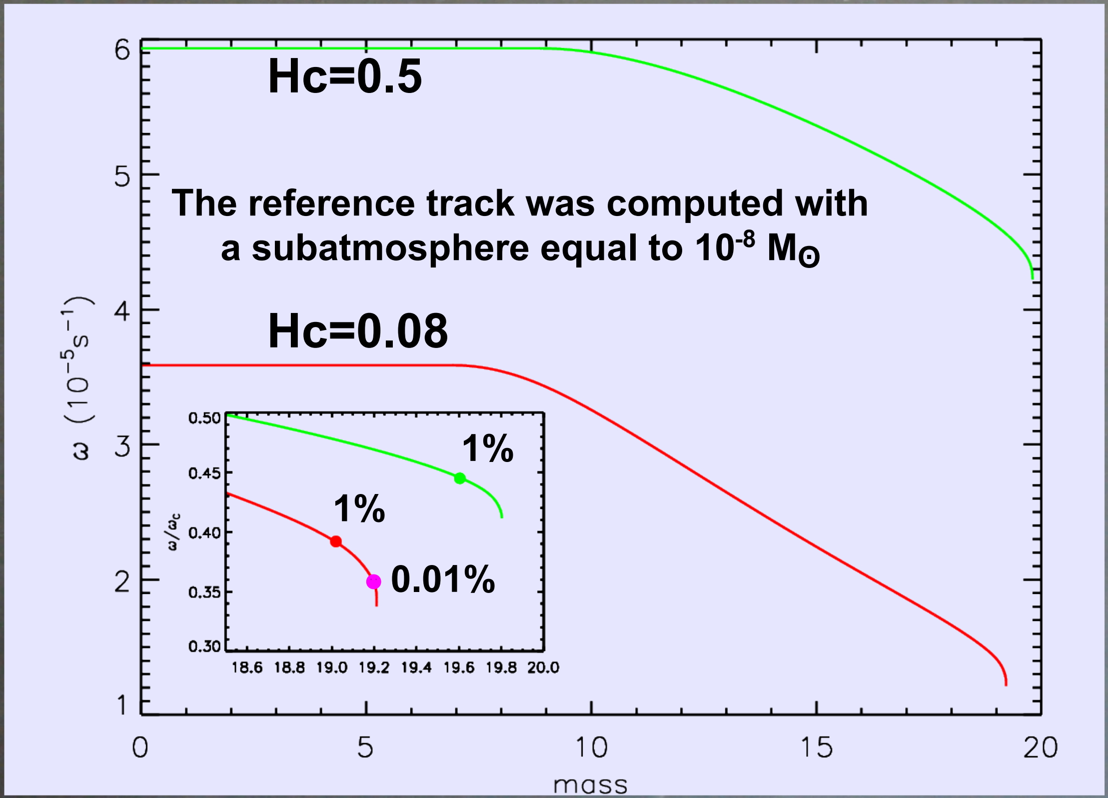
The same amount from each layer
The same percentage from each layer
An amount proportional to the distance from the surface
Down to a specific mass location or not
It is really correct to extract more angular momentum than that included in the mass lost?



Just a couple of additional *technical* problems...

Which mass size should be adopted in the subatmosphere?

1%
0.1%
0.01%
even less?



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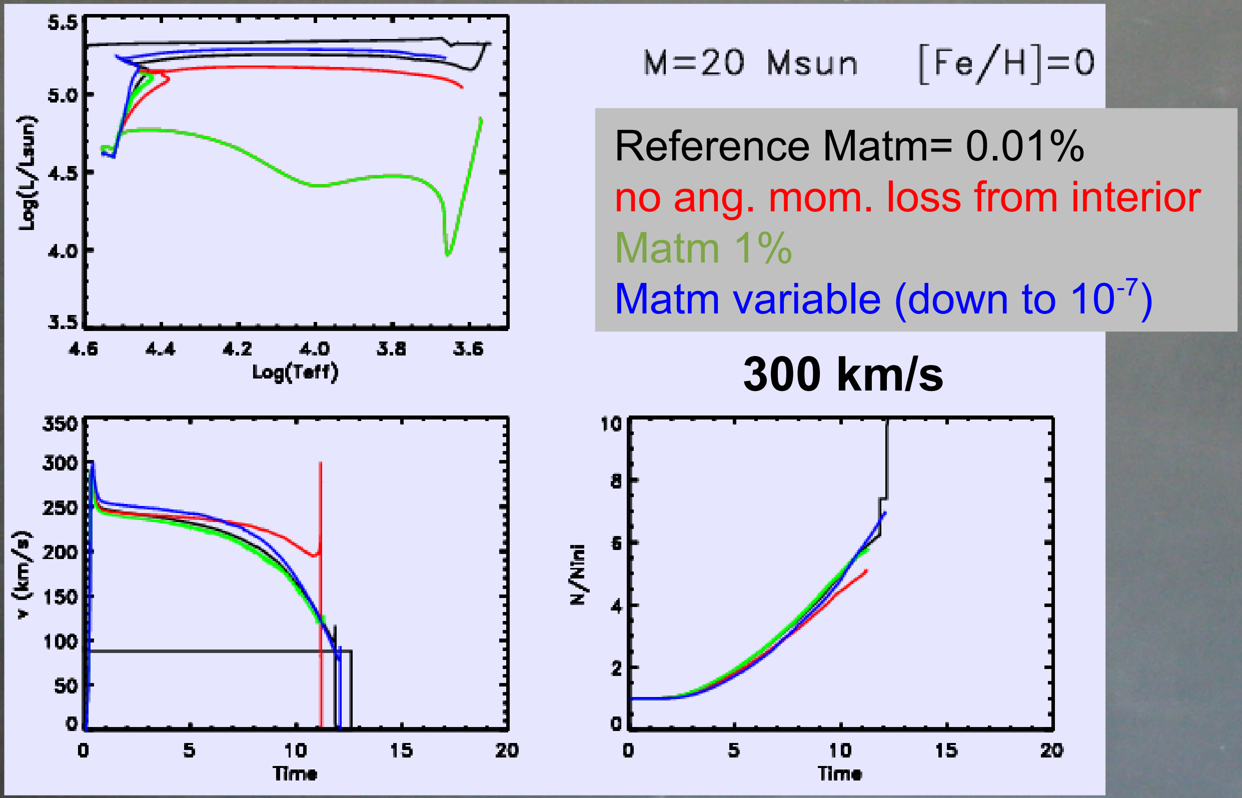
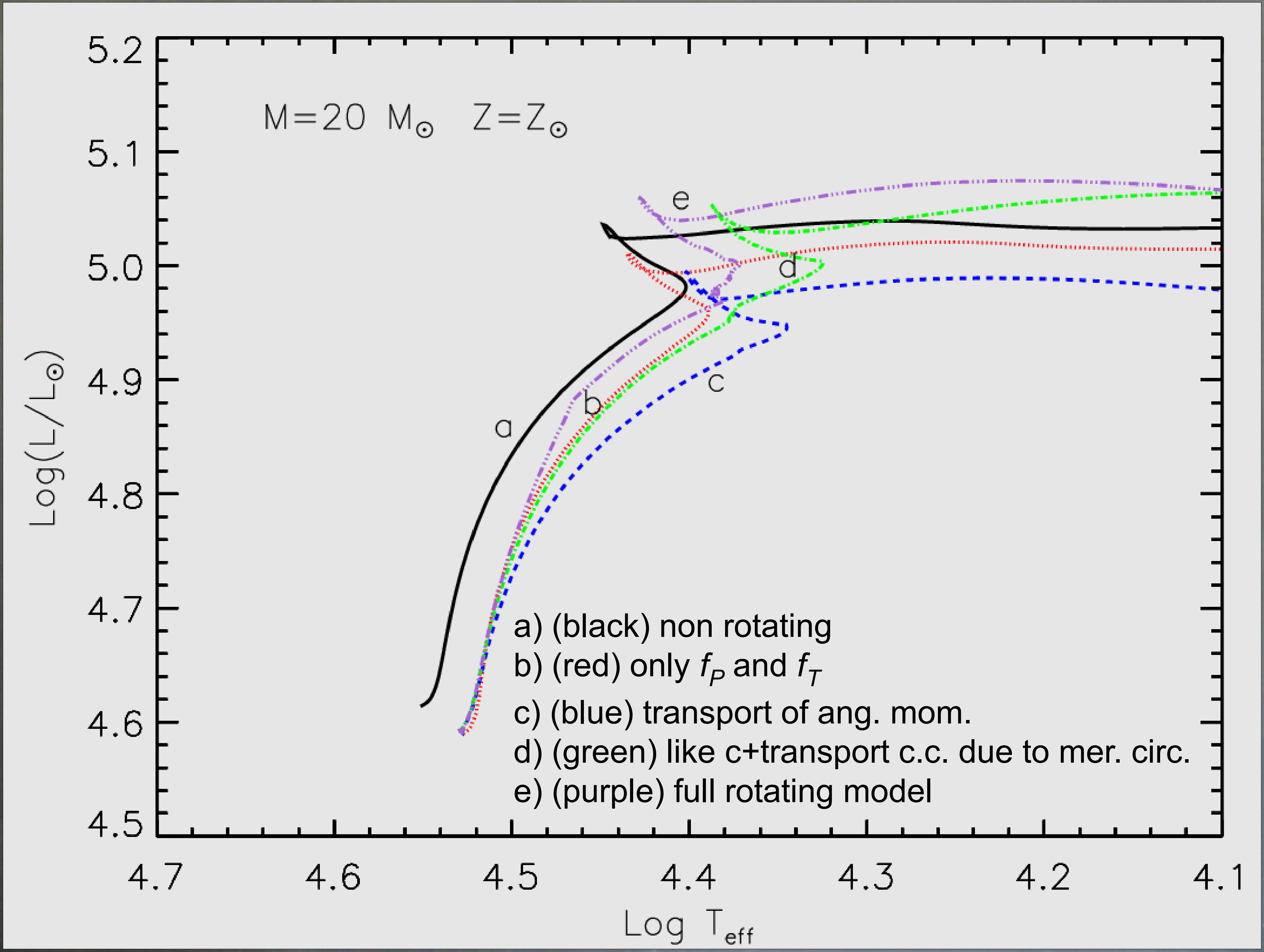


If you still think that it is useful to study the influence of rotation on the evolution of a star with these physical/numerical tools, in the sense that we can really learn something...

...you are *hopeless* but...

...let's go on...

Main effects of rotation on the surface properties of a star in H burning



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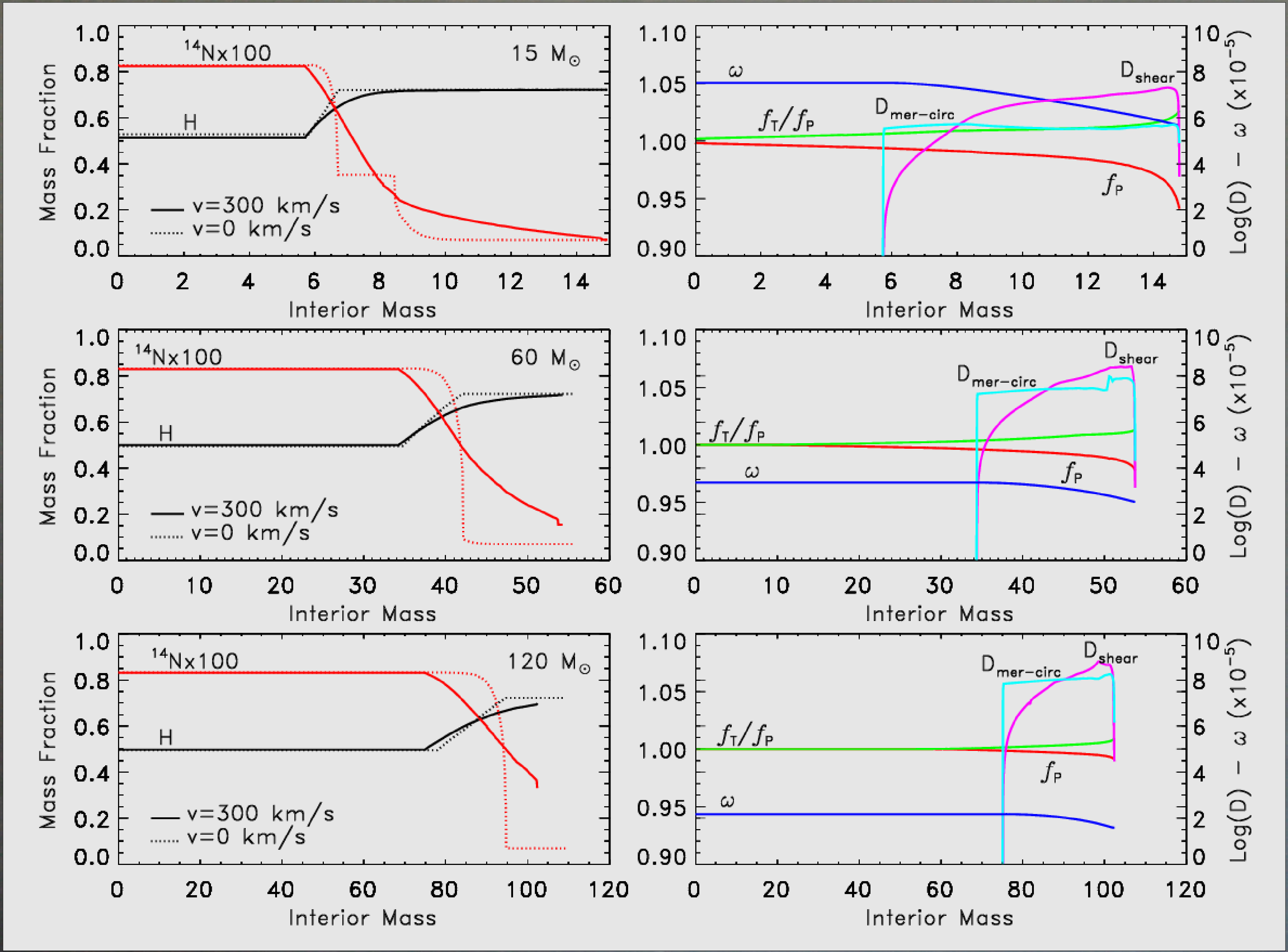
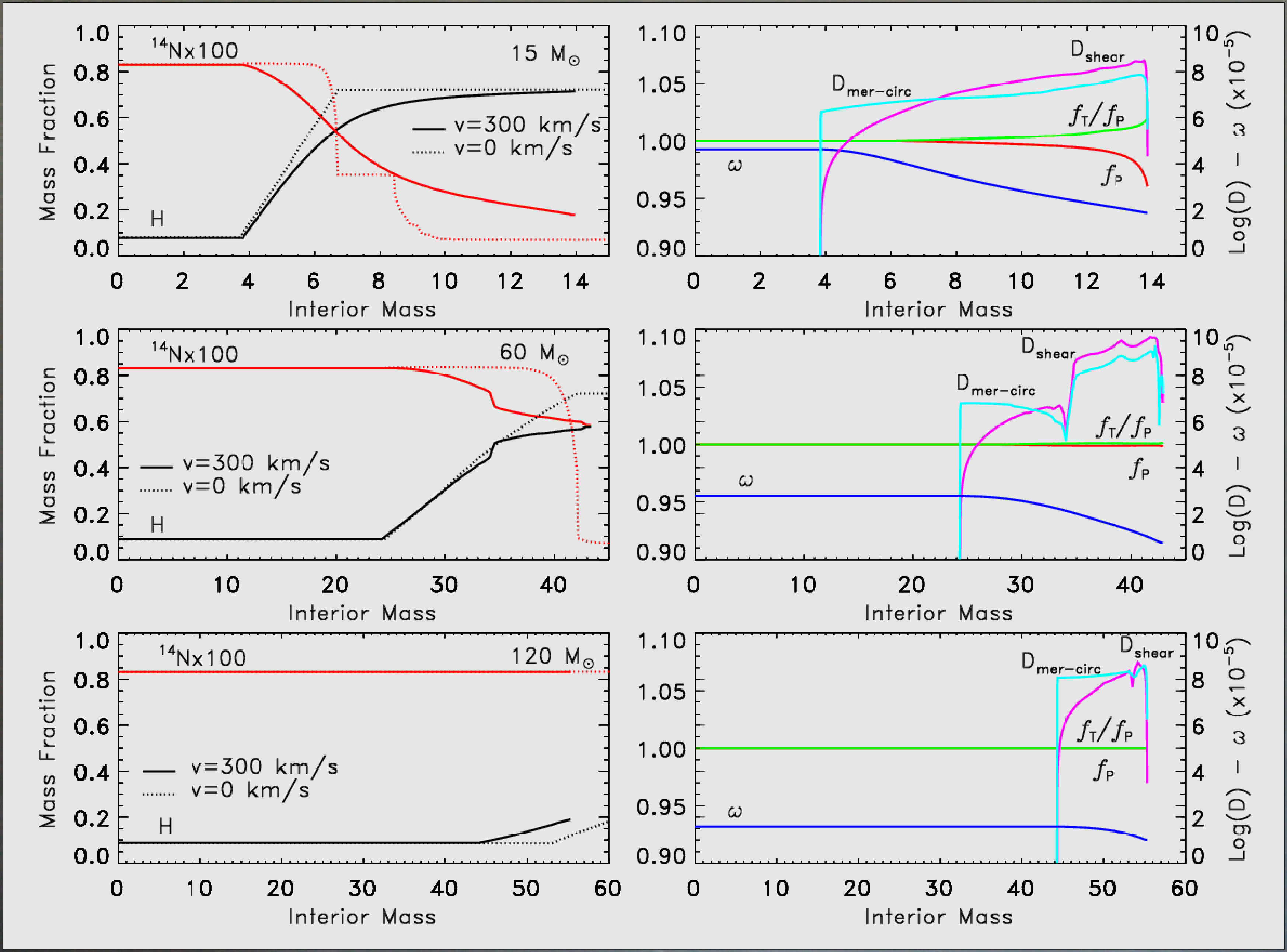
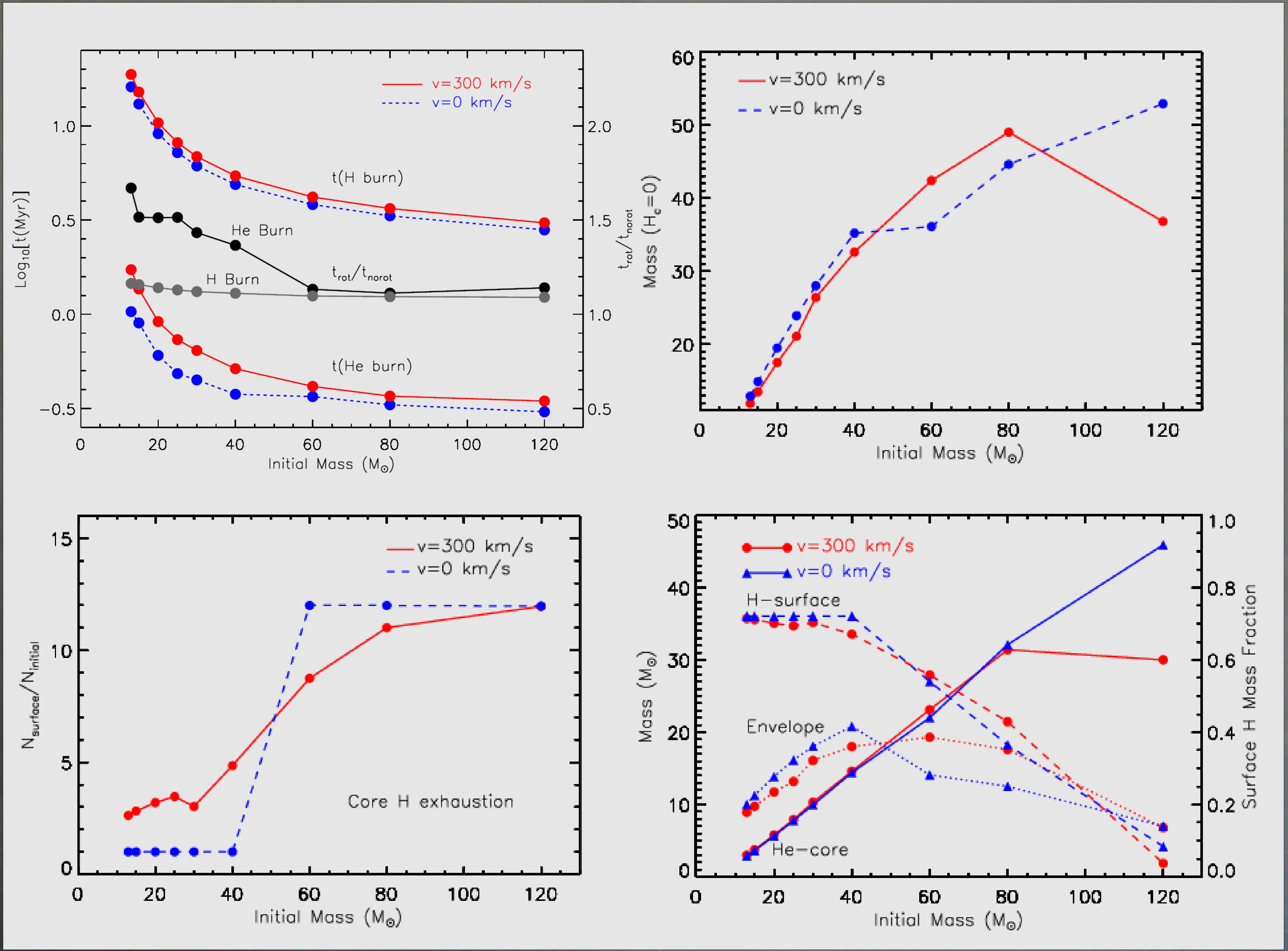


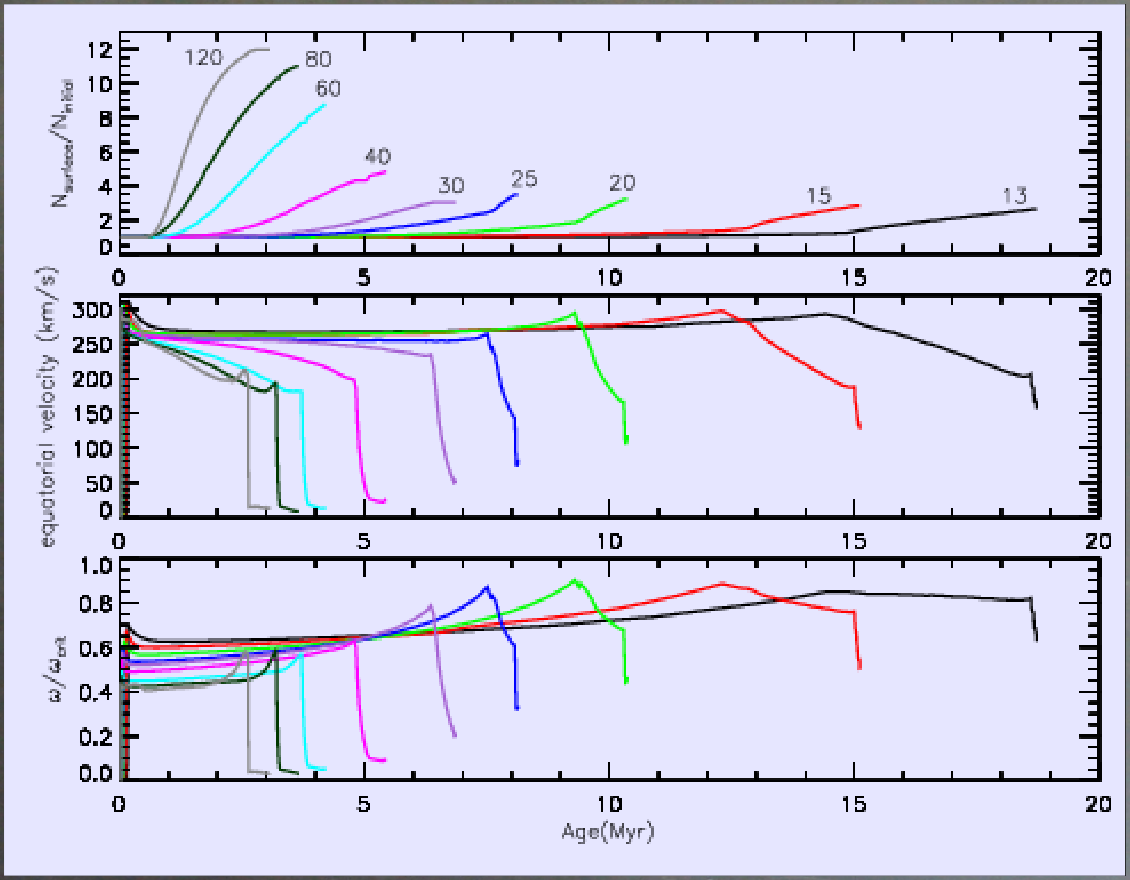
Main effects of rotation on the surface properties of a star in H burning

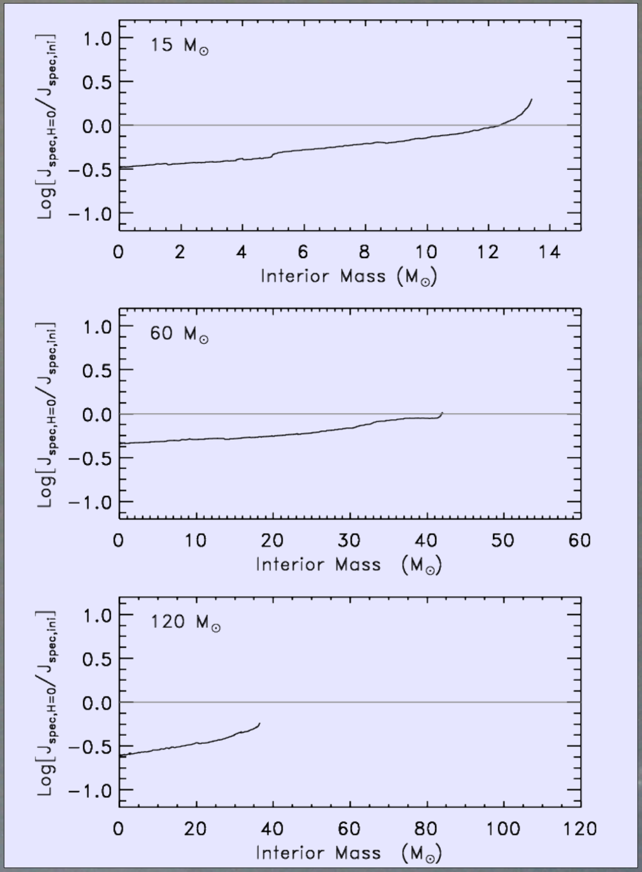


Global effects of rotation on the evolution of the massive stars in H burning

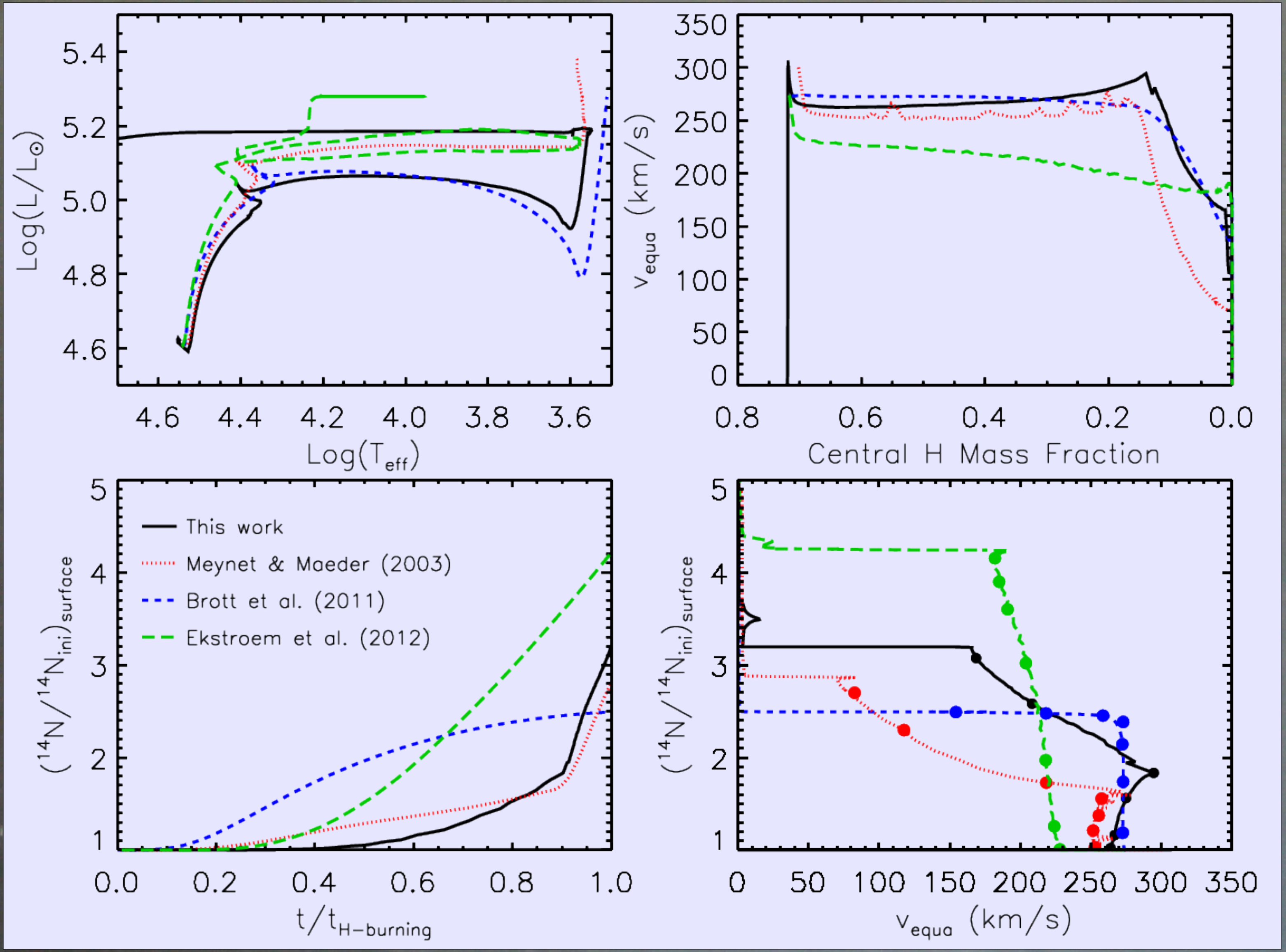


Global effects of rotation on the evolution of the massive stars in H burning





Comparison among different authors



Summarizing...at the end of the central H burning phase:

Models rotating at 300 km/s have:

smaller envelope masses but similar He core masses

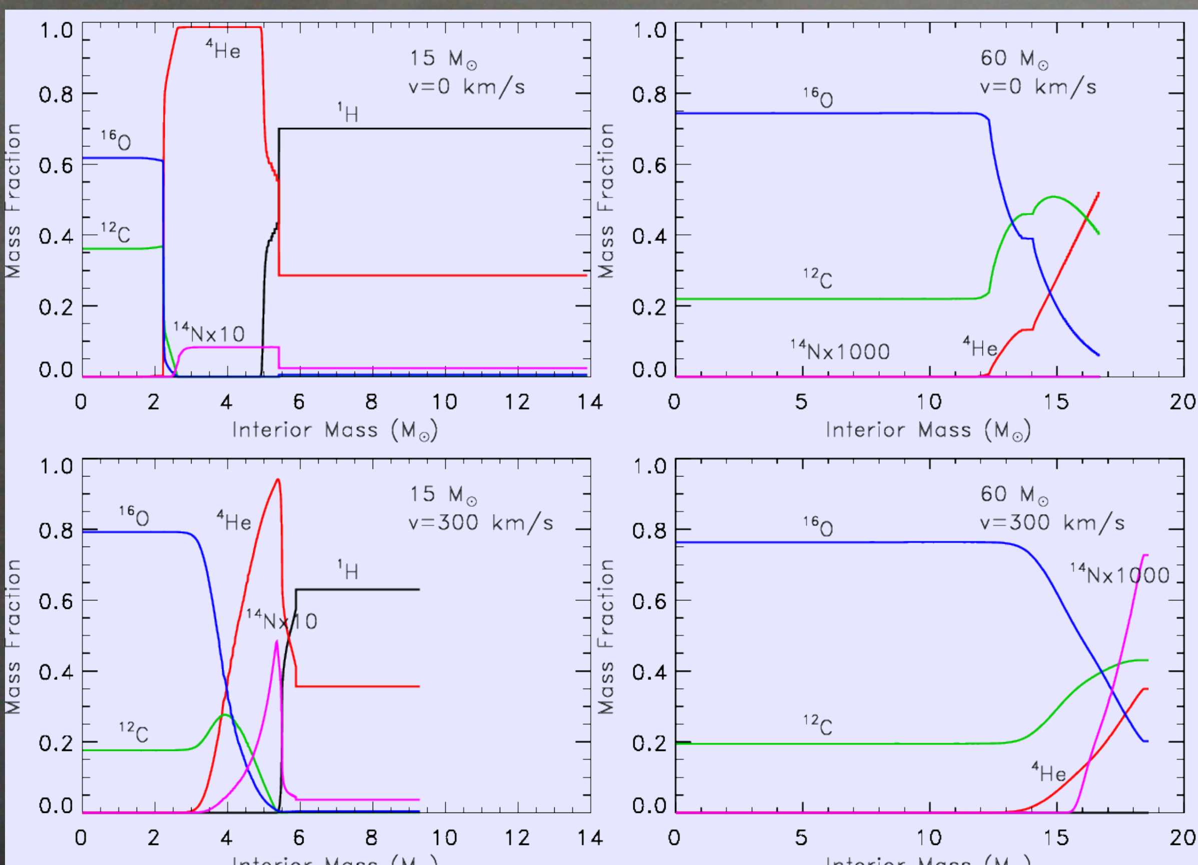
higher mean molecular weight in the envelope

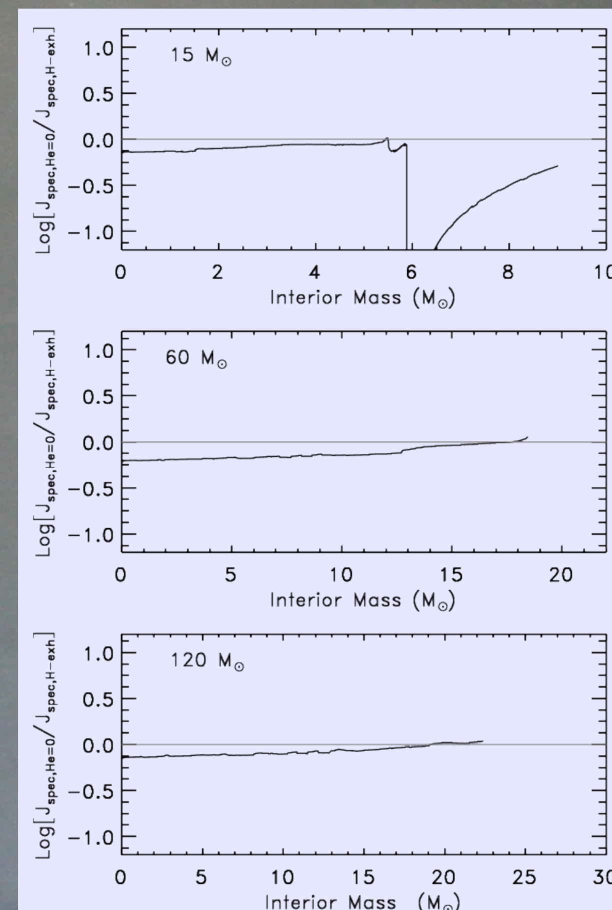
modified surface chemical composition

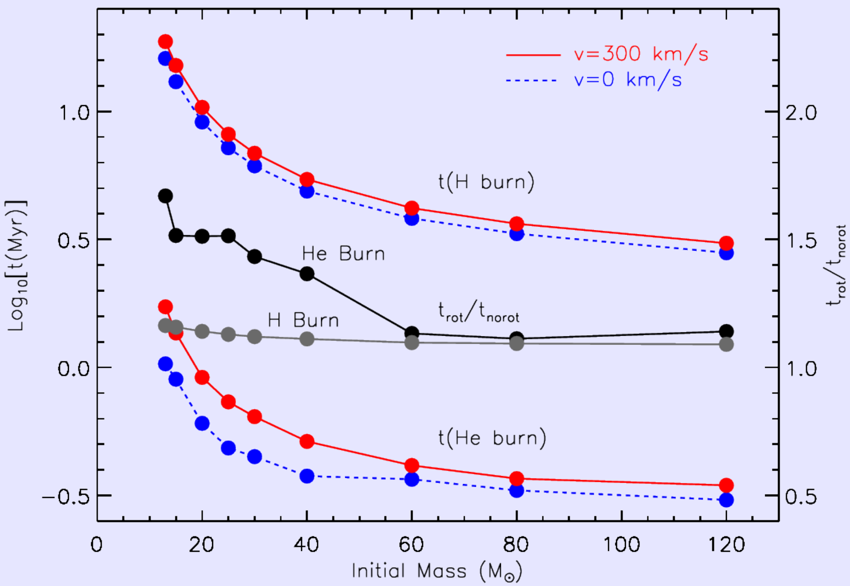
What happens to these stars (i.e. rotating initially at 300 km/s) in He burning?

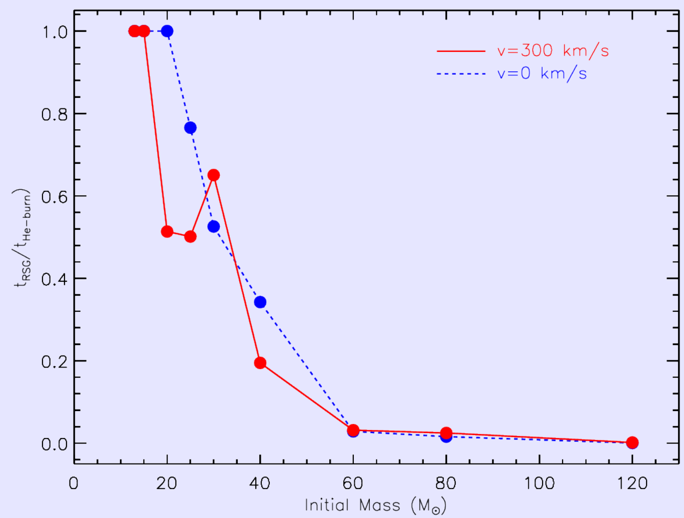
Rotation affects the further evolution of these stars in two ways:

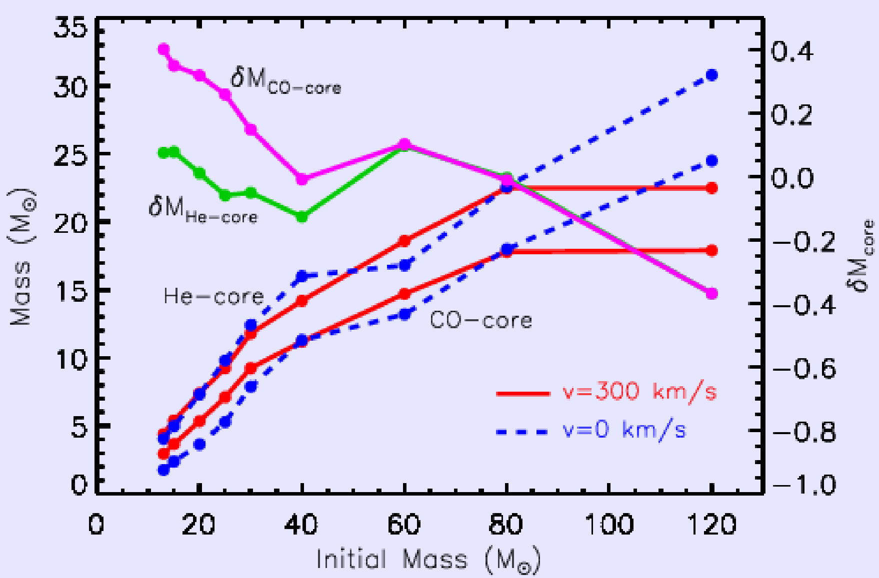
first of all *indirectly* because of the differences in the structures at the He ignition
second *directly*, basically within the He core:

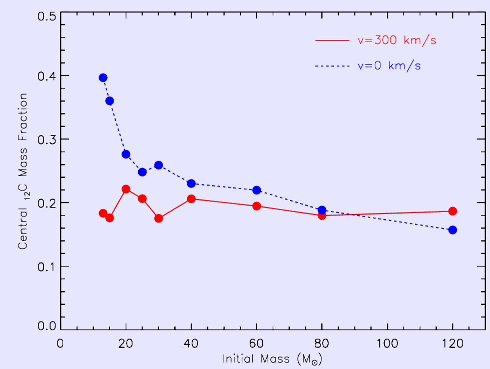


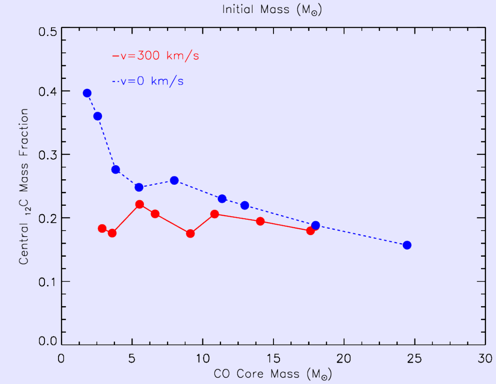



v=300 km/s
v=0 km/s
t(H burn)
He Burn
H Burn
t_rot/t_norot
t(He burn)
Log10[t(Myr)]
t_rot/t_norot
Initial Mass (M⊙)


v=300 km/s
v=0 km/s
t_RSG/t_He-burn
Initial Mass (M⊙)


δM_CO-core
δM_He-core
He-core
CO-core
v=300 km/s
v=0 km/s
Mass (M⊙)
δM_core
Initial Mass (M⊙)


v=300 km/s
v=0 km/s
Central 12C Mass Fraction
Initial Mass (M⊙)


v=300 km/s
v=0 km/s
Central 12C Mass Fraction
CO Core Mass (M⊙)

Summarizing again... models rotating at 300 km/s...
...show up at the beginning of the advanced phases with:

larger CO core masses

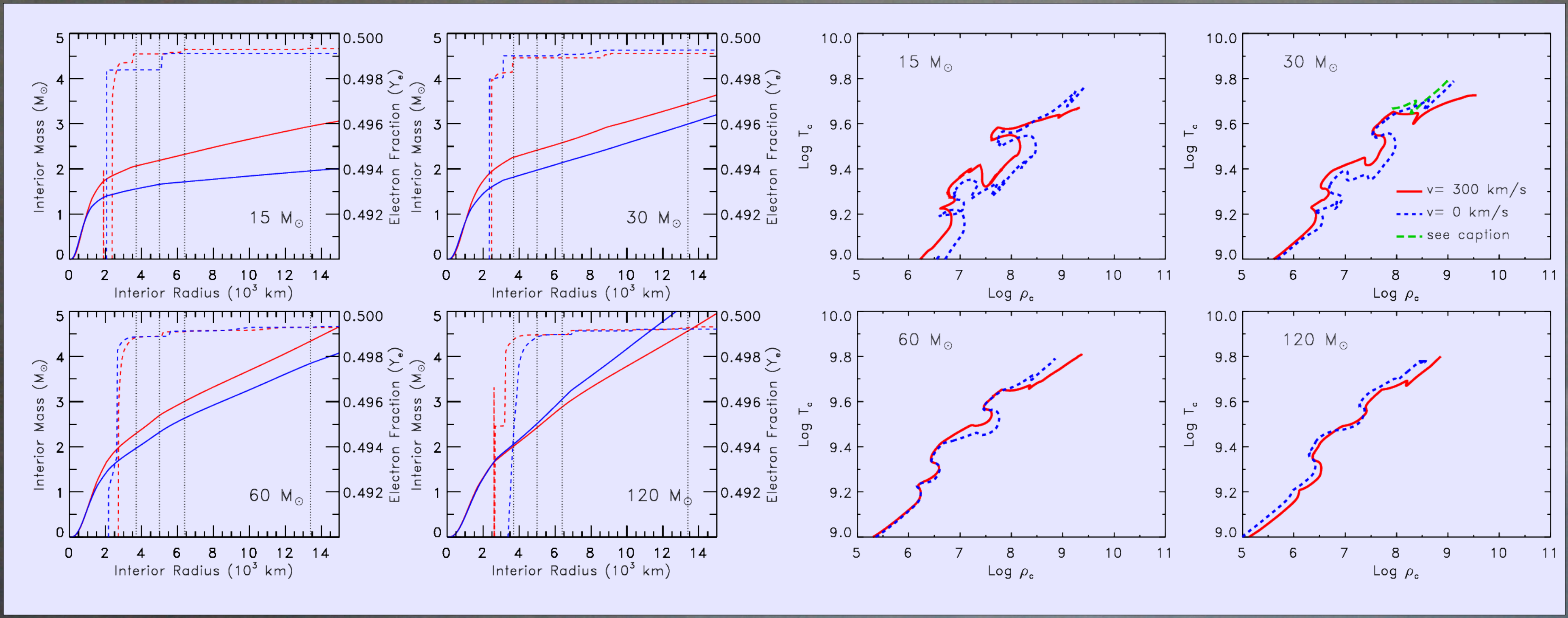
lower C/O ratio in the CO core

smaller total masses

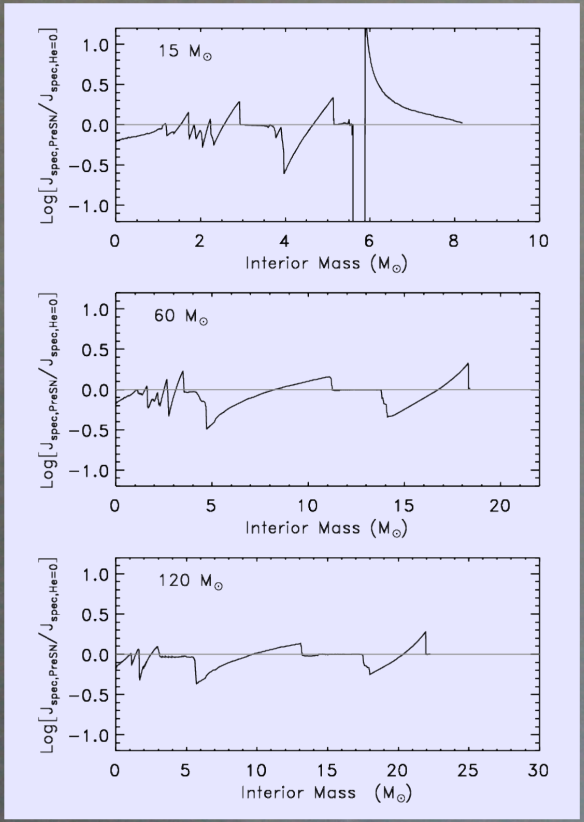
Hence...

rotating models will behave in the advanced burning phases basically

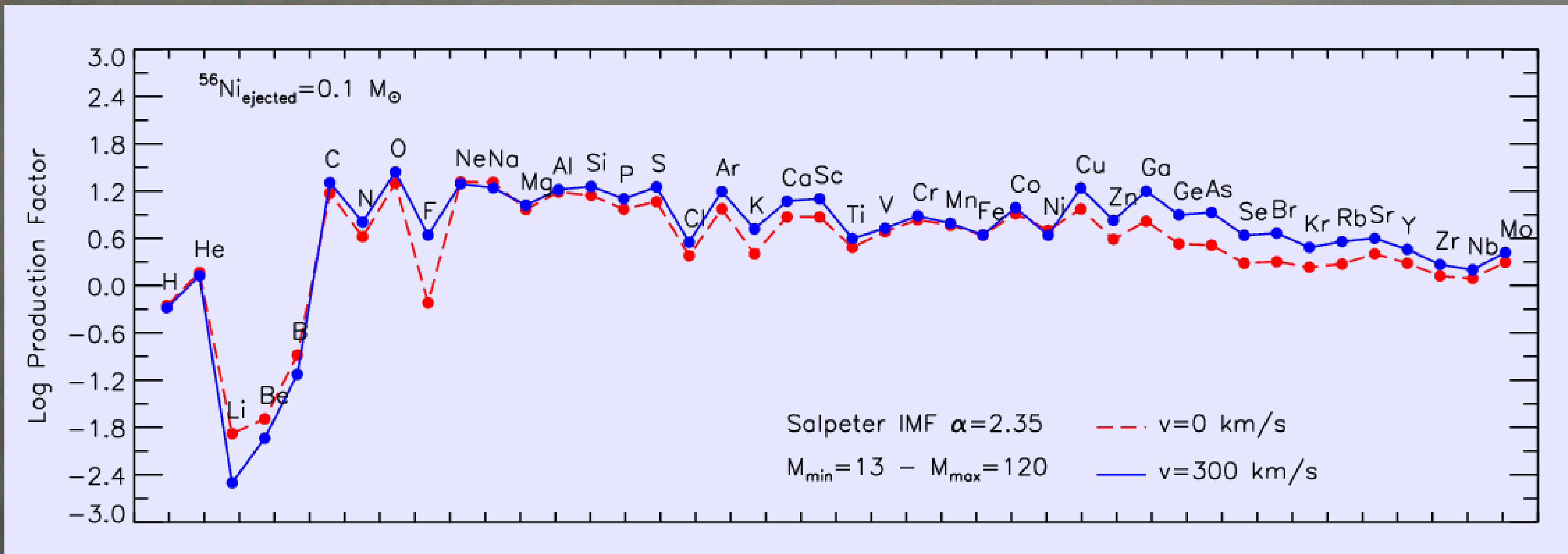
as more massive non rotating stars



There is no more time for the transport of angular momentum so the only changes occur in the convective zones where we assume instantaneous redistribution of the angular momentum so that ω is flat.



What about the yields?



Mild increase of the weak s-process component and of F

CONCLUSIONS
(personal but strong)

In order to make any meaningful comparison with the real stars it is mandatory to use a full set of models computed with a reasonable range of initial rotational velocities: a properly done population synthesis is necessary.

(the use of just an average velocity may be highly misleading)

The idea of specific transition masses, for example the limiting mass the explodes as a Type IIP supernova or the lowest mass that becomes a W_{CO} star must be dropped. It becomes meaningless in presence of rotation because rotation implies a SPREAD of these limiting masses over a certain range that depends on the initial distribution of the rotational velocities.

Since the inclusion of the effects of rotation on the evolution of a star is still *HIGHLY* qualitative, any statement suggesting the *necessity* to add some other phenomenon "because rotation can't reproduce some observable" is really premature: the first thing one should consider in this case is simply that rotation has been included in such a qualitiative way that it can't have a real predictive power. The present situation is totally similar to what happens with convection.

CONCLUSIONS
(more canonical)

Increase of the H burning lifetime

Modification of the surface chemical composition

Similar He core masses

Smaller envelope masses

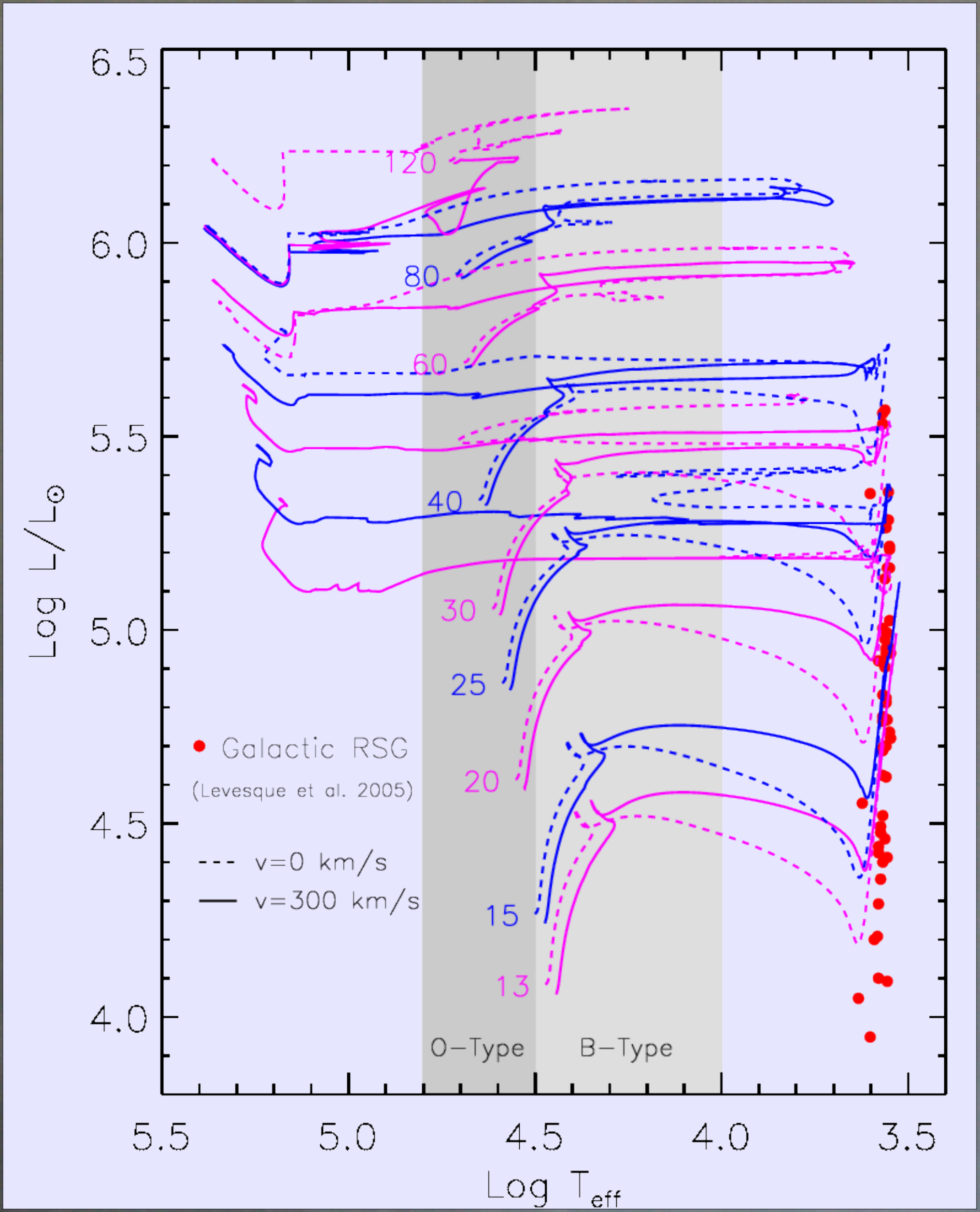
Larger number of WR stars + changes in the internal ratios among the WR subclasses

Larger CO core masses

Lower C/O ratios at the end of the central H burning

Final steeper M-R relation

More massive remnants for a fixed final kinetic energy of the ejecta



6.5
6.0
5.5
5.0
4.5
4.0
Log L/L⊙
5.5
5.0
4.5
4.0
3.5
Log Teff
120
80
60
40
30
25
20
15
13
Galactic RSG
(Levesque et al. 2005)
v=0 km/s
v=300 km/s
O−Type
B−Type