Using quantum mechanics to understand the life cycles of stars

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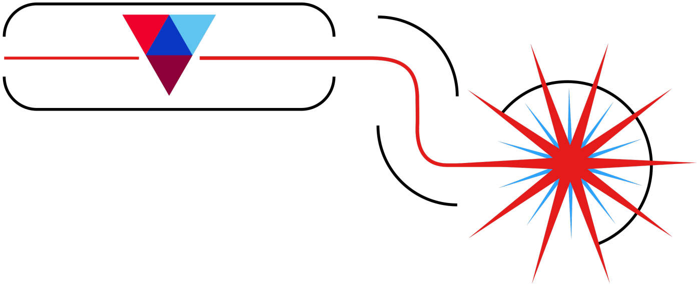
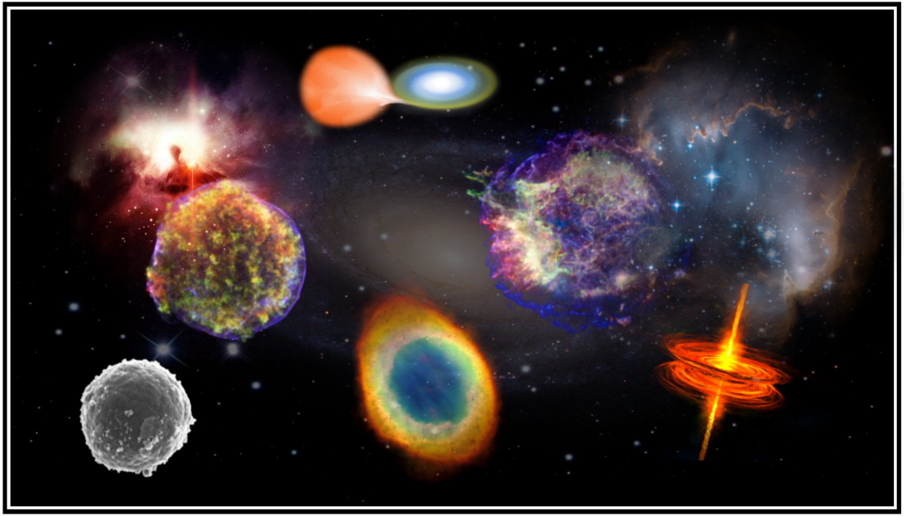


Getting from this. . .

. . .to this



Figures courtesy of NASA APOD (apod.nasa.gov)

Outline

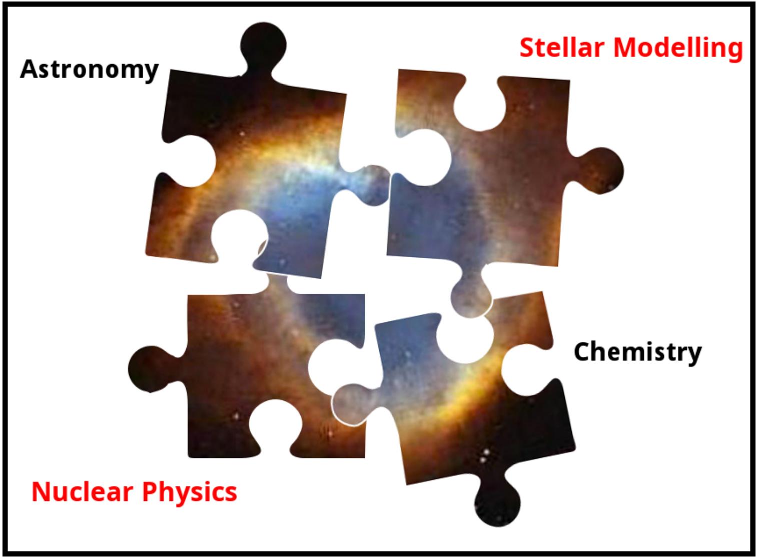
1. Introduction to Nuclear Astrophysics

2. Nuclear Reaction Cross Sections

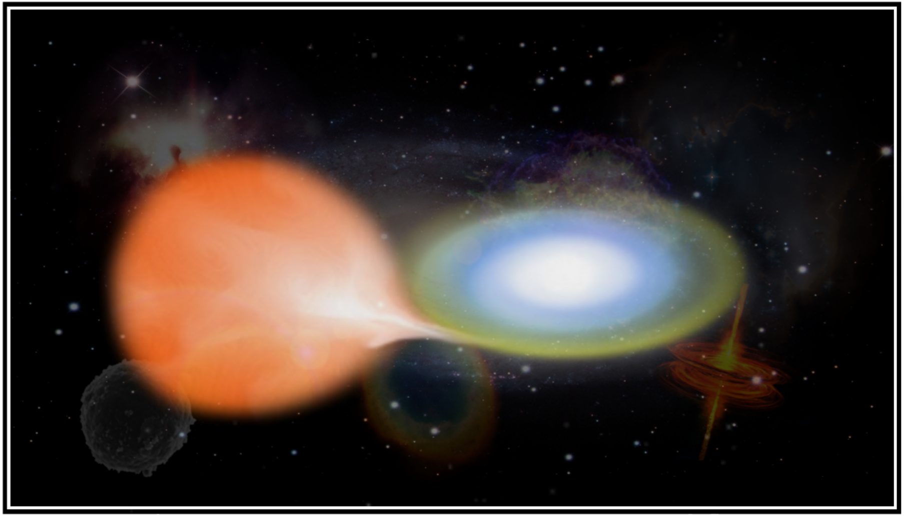
3. Nuclear Scattering Experiments

4. An Example: ^{34}S Destruction in Novae

Tools to understand the stars



An Example: Novae



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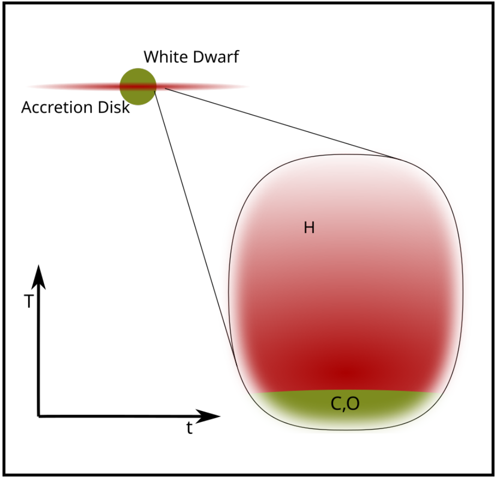
An Example: Classical Novae

- Occur in binary systems (Catalclysmic variables)



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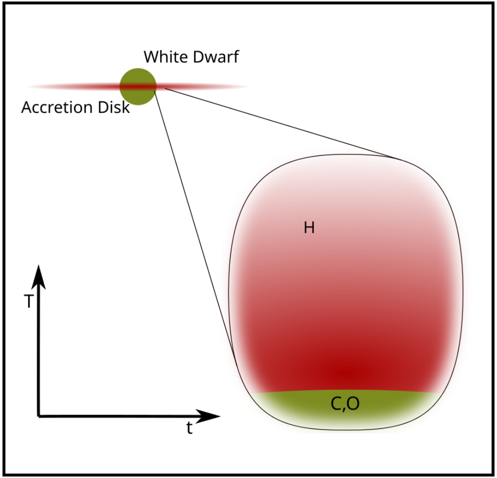
- Material accreted onto white dwarf
- Base of accreted matter becomes electron-degenerate



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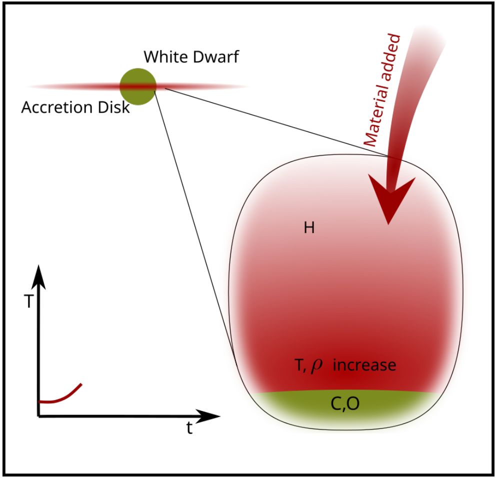
- Temperature increases $\rightarrow$ thermonuclear runaway



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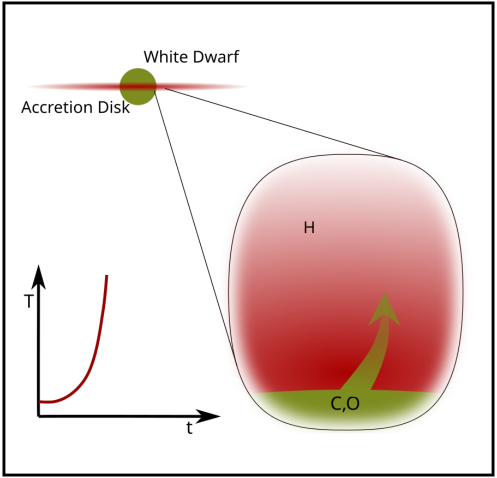

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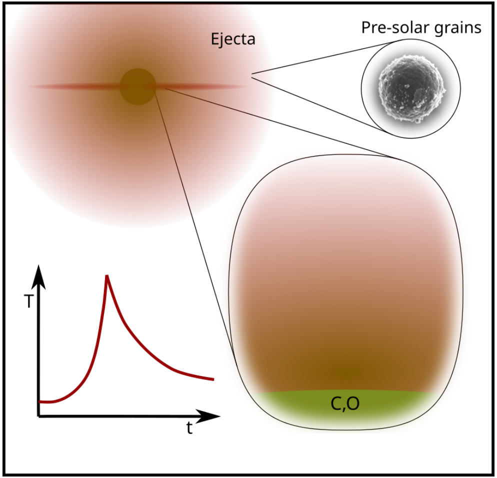
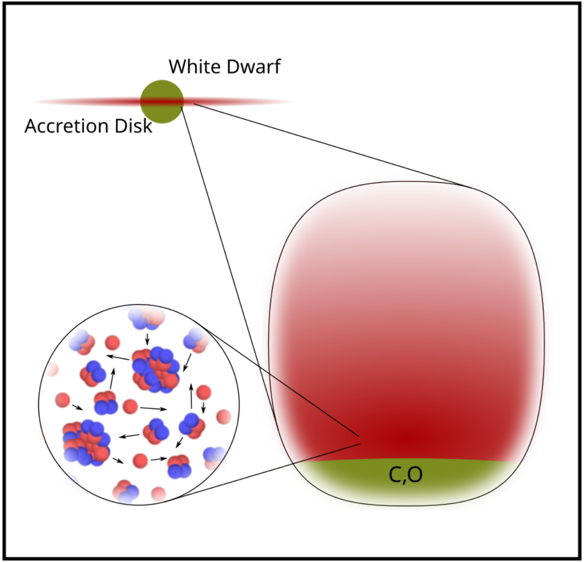
- Nuclear reactions occur
- Temperature increases $\rightarrow$ thermonuclear runaway
- White dwarf material dredged into burning region
- Thermonuclear explosion $\rightarrow$ (total?) ejection of matter

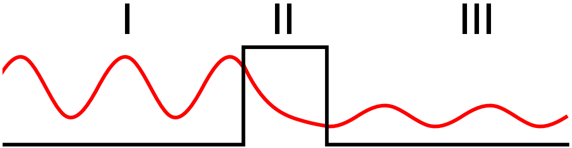




Reaction Rate per Particle Pair

$$\langle \sigma v \rangle \propto \int_0^\infty E\sigma(E)e^{-E/kT}dE$$

- Particle velocities distributed according to a Maxwell-Boltzmann distribution
- For a known temperature, we only need to know $\sigma(E)$ to calculate reaction rate



$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 < x < a \\ 0 & x > a \end{cases}$$

$$\Psi_I = Ae^{ik_1 x} + Be^{-ik_1 x}$$
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$$k_1 = k_3 = \sqrt{\frac{2mE}{\hbar^2}}$$
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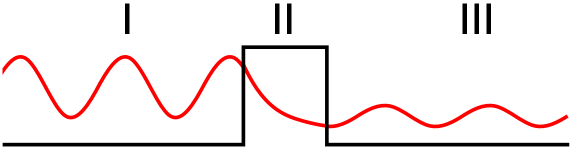
Quantum Tunneling



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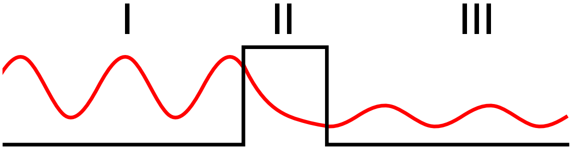

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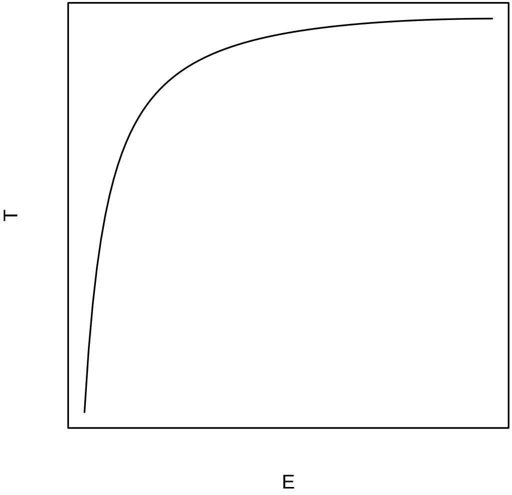
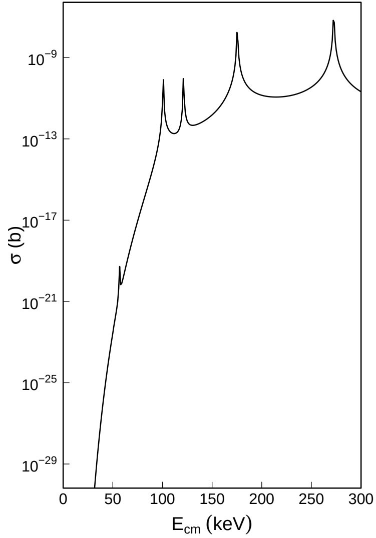
- F = 0
- k_2 is imaginary if $E < V_0$
- Wave function in region II becomes

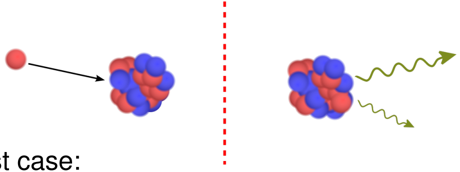
$$\Psi_{II} = De^{-k_2 x}, \quad k_2 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

Probability of tunneling:
$T = |E|^2/|A|^2$

$$T = \frac{1}{1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)}\sinh^2(k_2 a)}$$

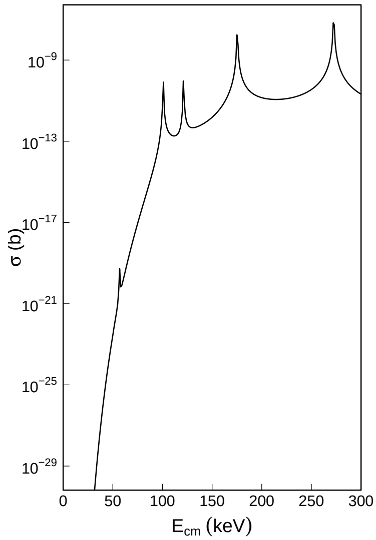


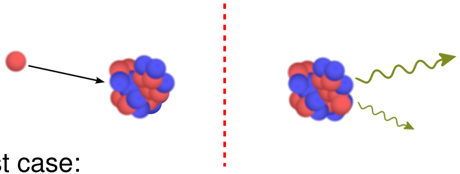




- Simplest case:
  - Bombard target with a beam of particles (^{34}S+p)
  - Count the γ rays

Measuring Nuclear Reaction Cross Sections





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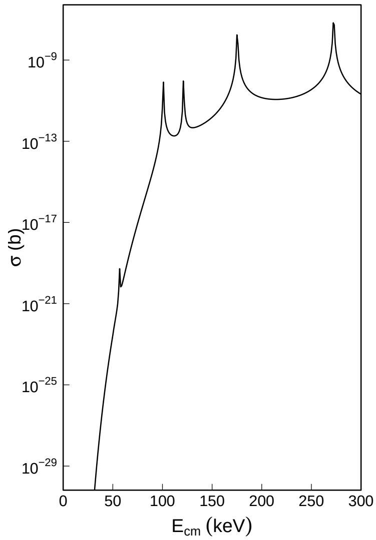
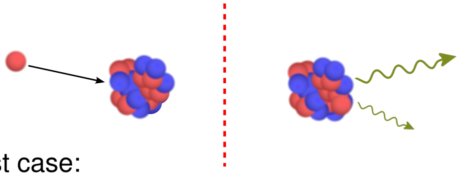
- At 300 MK, $k_B T = 26 keV$!
- So only highest energy particles undergo nuclear reactions





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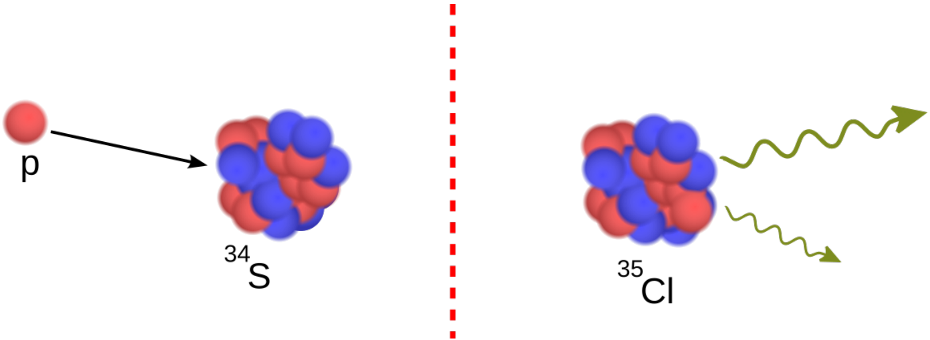
- Consider ^{34}S(p,γ)^{35}Cl
  - Measure resonances at $E_r^{cm} \approx 100$ keV
  - Typical γ-ray detector efficiency $\sim$1%
  - Assume best-case solid ^{34}S target
  - Count rate: 1×10^{-7} counts per Coulomb
  - World's most intense proton beam: 20 mA: 1×10^{-4} counts per hour

10 counts in 11 years

Particle Transfer Reactions



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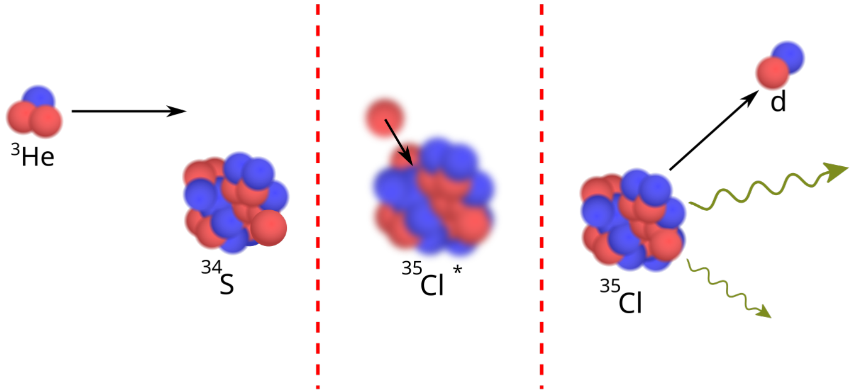
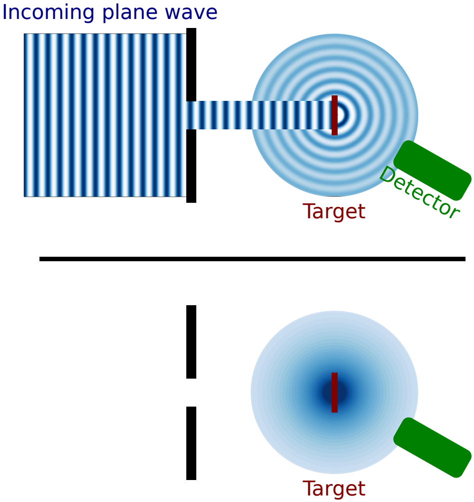


Elastic Scattering



Case for $\ell = 0$

- Total wave function far from the target

$$\Psi(\vec{r}) = N\left[e^{i\vec{k}\cdot\vec{r}} + f(\theta)\frac{e^{ikr}}{r}\right]$$

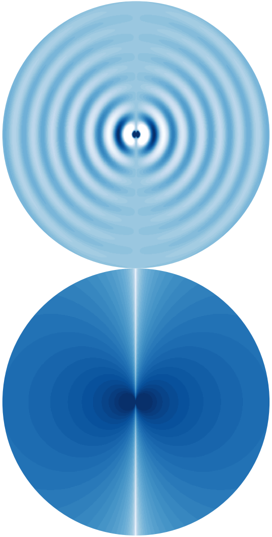
- Detectors don't measure wave functions, they measure *flux* $= v\Psi^*\Psi$
- To combine the coordinate systems, we expand the incoming wave in terms of partial waves:

$$e^{ikz} = \sum_{\ell=0}^{\infty}(2\ell+1)i^{\ell}j_{\ell}(kr)P_{\ell}(\cos\theta)$$

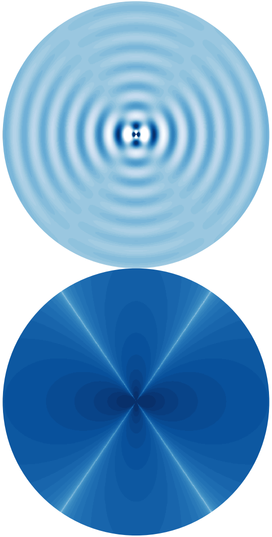
- The scattering amplitude becomes:

$$f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)$$

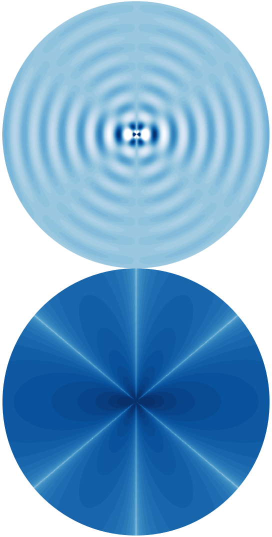
Elastic Scattering



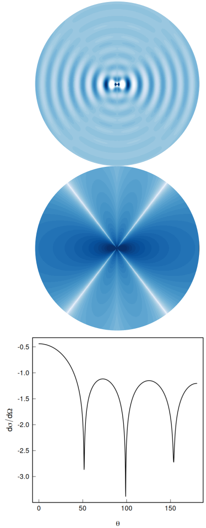
Case for $\ell = 1$



Case for $\ell = 2$



Case for $\ell = 3$



Case for $\ell = 0, 1, 2, 3$

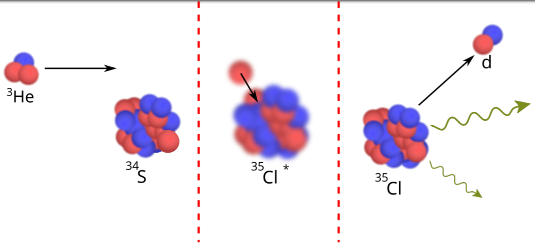
- The scattering amplitude is:

$$f(\theta) = \frac{1}{k}\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)$$

- Cross section measured by detector:

$$\frac{d\sigma}{d\Omega} = f^*(\theta)f(\theta)$$

- Traditionally, use cross section to find δ_ℓ, which derive from the shape of the nuclear potential



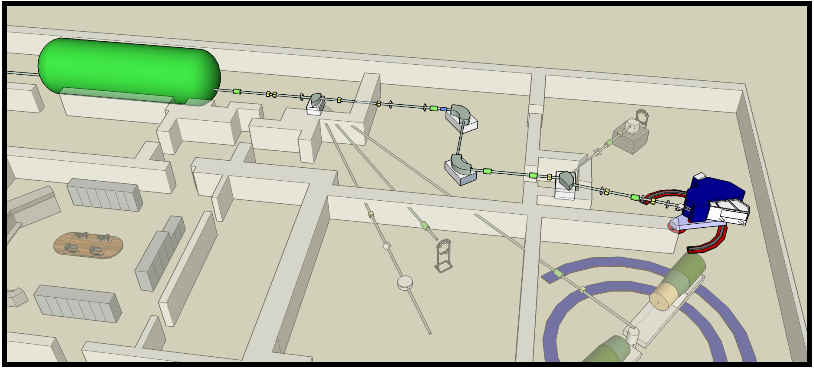
- Elastic scattering on previous slides is dominant
- Small perturbation causes direct transfer from initial state to final state
- Cross section:

$$d\sigma/d\Omega \propto |\langle\Psi_f^*|V|\Psi_i\rangle|^2$$

- Use partial wave expansion again, but now there are often only a few contributing terms $\rightarrow d\sigma/d\Omega$ tells us ℓ!
- Final state is described by core (^{32}S) plus transferred nucleons
  - Projectile in optical potential of target
  - Perturbing nuclear potential (including transfer of nucleons)
  - Outgoing particle in potential of residual

- Four-university consortium
  - North Carolina State University
  - North Carolina Central University
  - The University of North Carolina at Chapel Hill
  - Duke University
- Three accelerator facilities
  - The Tandem accelerator laboratory
  - The Laboratory for Experimental Nuclear Astrophysics
  - The High Intensity γ-ray Source

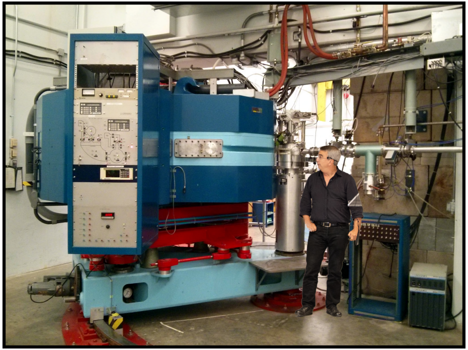
The Triangle Universities Nuclear Laboratory

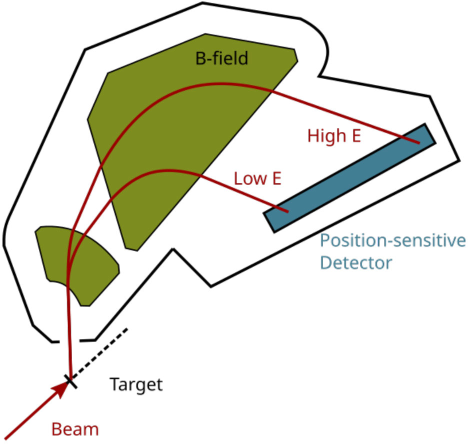


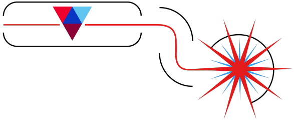
- Beam capabilities
  - p, d ($\sim 1\mu$A)
  - ^{3}He, ^{4}He ($\sim$ 500 enA)
  - Heavier species with SNICS (^{7}Li at $\sim$ 400 enA)
  - Chopping/Bunching capabilities
- 10 MV Tandem accelerator
- Enge Split-pole Spectrograph
  - Placed on the high-resolution beam-line at TUNL

Performing transfer measurements

- Requirements
  - Measure outgoing (charged) particles
  - Angle dependent measurement
  - High resolution (tens of keV)
  - Particle ID







An Example: ^{34}S Destruction in Novae

PHYSICAL REVIEW C **99**, 055812 (2019)

Experimental study of ^{35}Cl excited states via ^{32}S(α, p)

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(Received 20 February 2019; published 30 May 2019)

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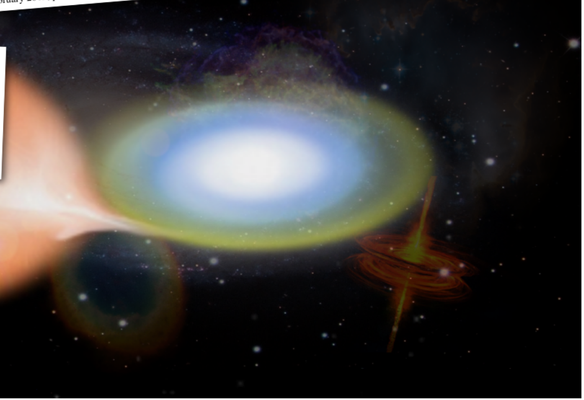

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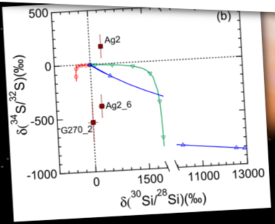

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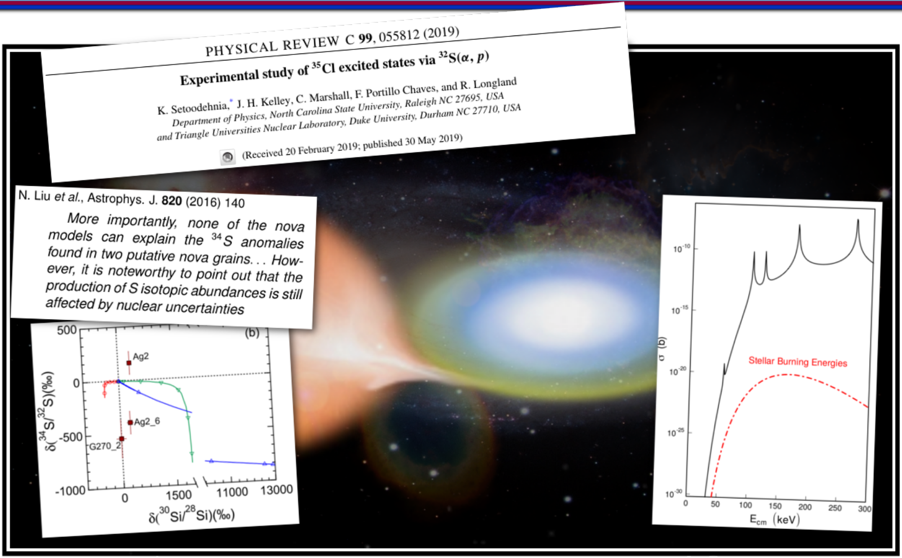

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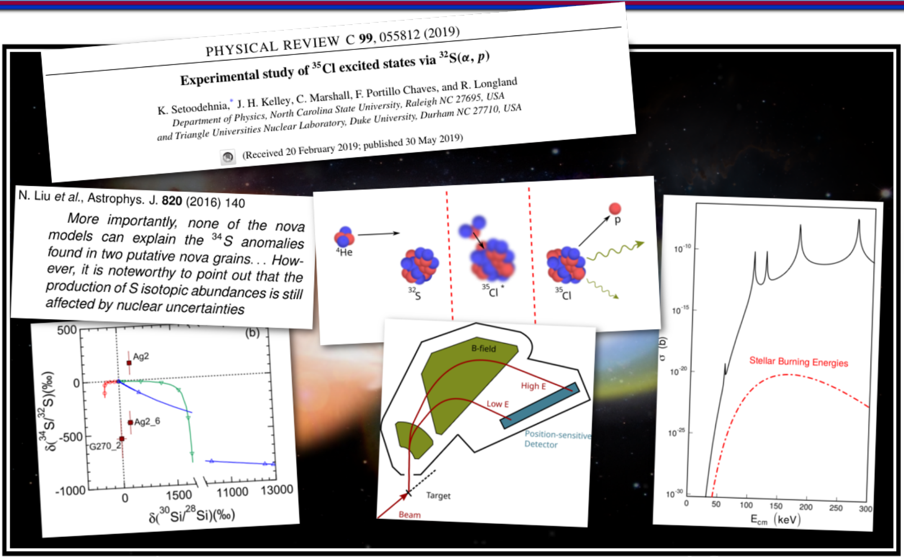
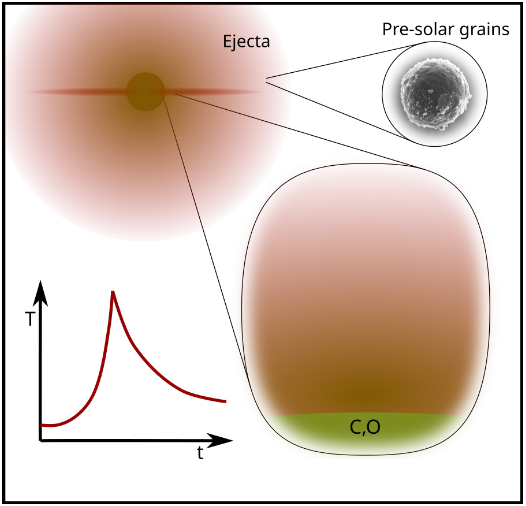
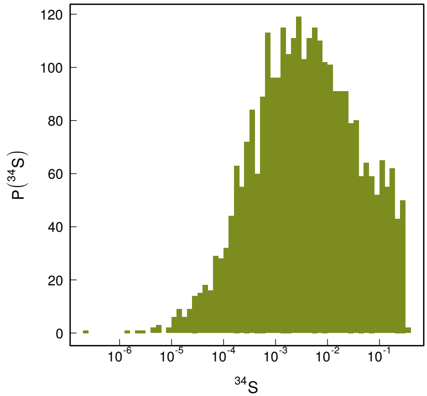


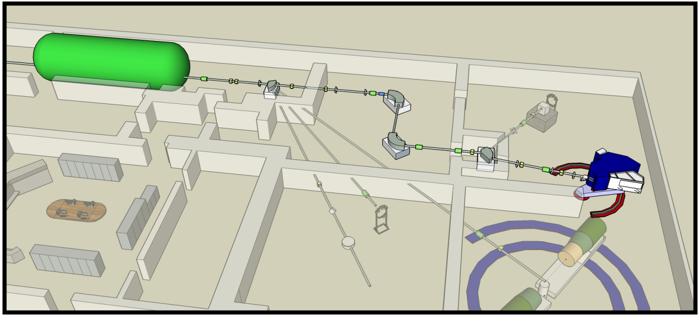
- Situation in 2016
  - $^{34}S(p,\gamma)^{35}Ar$ reaction rate purely theoretical
  - Assume "typical" uncertainties of a factor of 10
  - Simple nova model



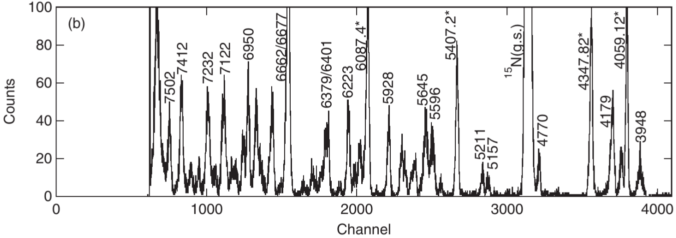


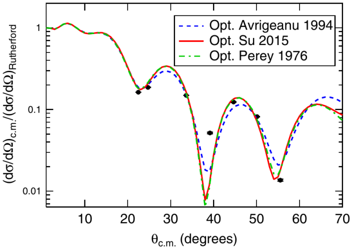
Nuclear uncertainties were so large that sulfur *could not* be a good diagnostic of novae

Experimental Details



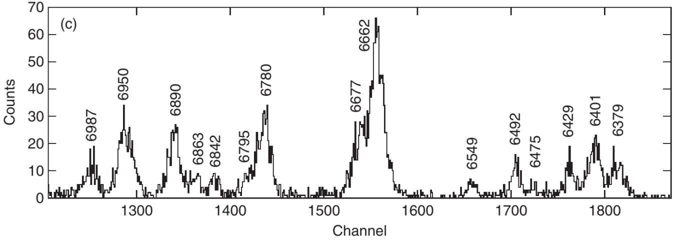
- Targets produced by evaporation of CdS onto 30 $\mu g/cm^2$ carbon foils (2000 Å)
- Rutherford Backscattering Spectrometry (RBS)
  - 16 (2) $\mu g/cm^2$ of sulphur
  - 44 (4) $\mu g/cm^2$ of cadmium
  - 32 (3) $\mu g/cm^2$ of carbon
- $^4He^{++}$ beam accelerated to 21 MeV
- Light reaction products measured at 10°, 15°, 19°, 30°, 35°, 40°, 45°, and 50°
- Positions of light reaction products yields excitation energies

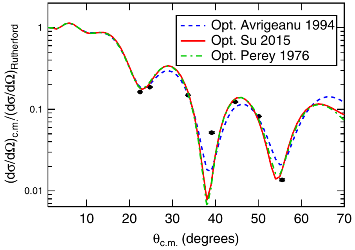




- 66 individual excited states populated
- 10 astrophysically-important states
- Excitation energies extracted
- Spin-parities of excited states inferred

Experimental Results





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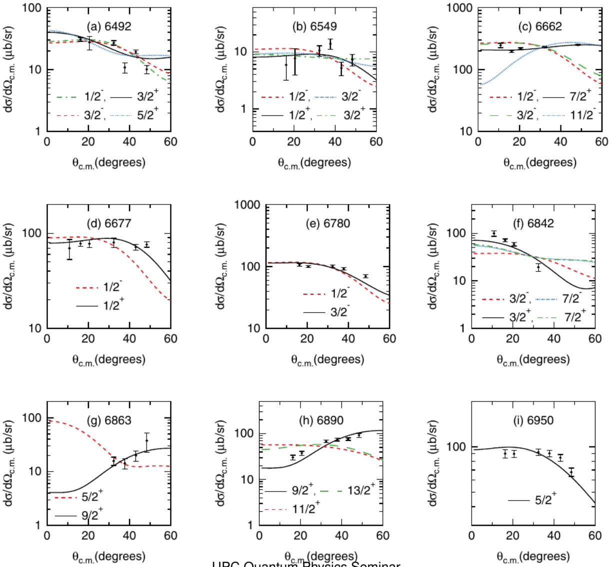
Spin-parity Assignments



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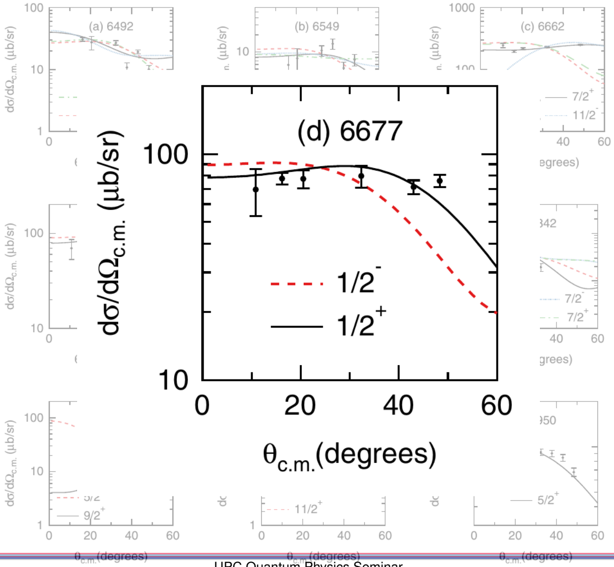
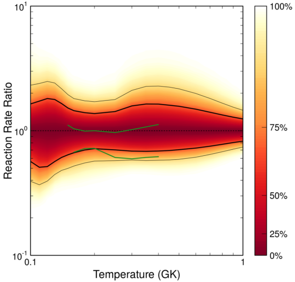
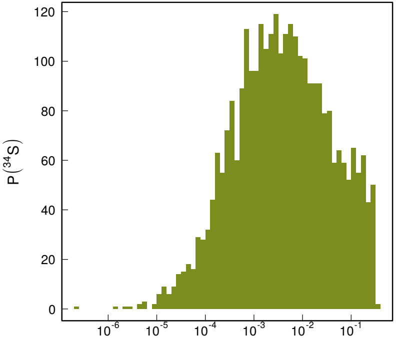
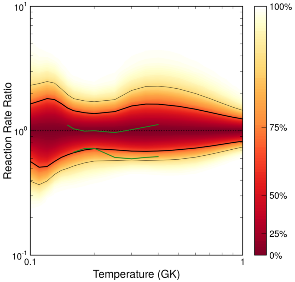




- Reaction rate uncertainties determined through Monte Carlo uncertainty propagation technique
- Reaction rate uncertainty *reduced* by an order of magnitude
- Gillespie. PRC **96** 025801

- Nucleosynthesis uncertainty of ^{34}S in novae dramatically reduced
- Identified states that most affect nucleosynthesis





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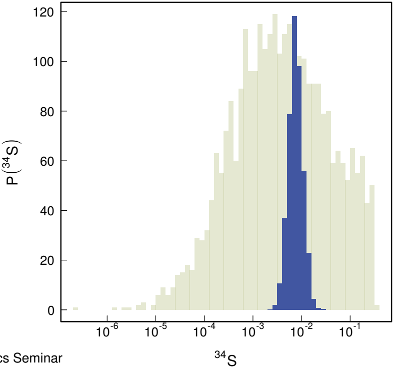
- Gillespie. PRC **96** 025801

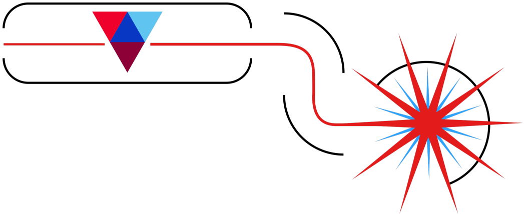
- Nucleosynthesis uncertainty of ^{34}S in novae dramatically reduced
- Identified states that most affect nucleosynthesis



Summary

- Astrophysically-important nuclear reaction cross sections are notoriously hard to measure
- Novel methods can be employed to constrain them
- At NC State and the Triangle Universities Nuclear Laboratory, we perform particle-transfer reactions using a high-resolution magnetic spectrograph

- ^{34}S(p,γ)^{35}Cl reaction rate uncertainty reduced by almost an order of magnitude
- Sulphur production in nova explosions significantly constrained
- Aspects of cross section identified for further study



Thank you!

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Let's calculate the rate of ^{34}S(p,γ)^{35}Cl in a nova explosion.

- $T_{\text{nova}} \sim 250$ MK
- $\rho_{\text{nova}} \sim 8 \times 10^3 g/cm^3$
- $M_{\text{nova}} \sim 0.04 M_{\odot} = 1 \times 10^{29}$ g

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- Initial mass fraction of ^{34}S $X(S) = 5 \times 10^{-6}$
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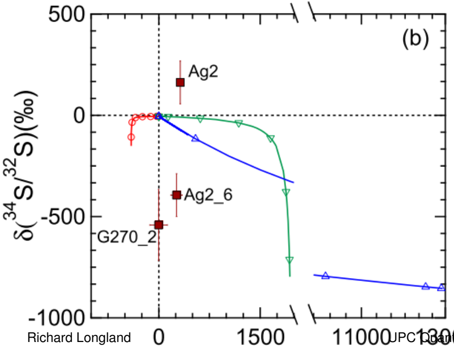
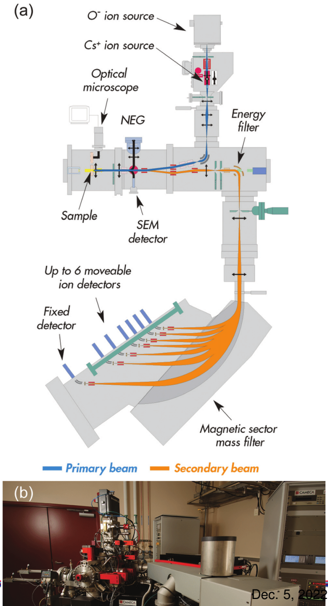
- $\langle\sigma v\rangle$ is the reaction rate per particle pair
- Rate of reactions per ^{34}S $\lambda_S \approx \rho \frac{X_H}{M_H} N_A\langle\sigma v\rangle \sim 6 \times 10^{-4} s^{-1}$
- $R = 4 \times 10^{18}$ reactions per second!

An Example: Classical Novae - Dust

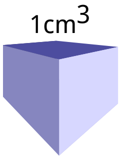
N. Liu *et al.*, Astrophys. J. **820** (2016) 140

> *More importantly, none of the nova models can explain the ^{34}S anomalies found in two putative nova grains... However, it is noteworthy to point out that the production of S isotopic abundances is still affected by nuclear uncertainties*





- What if we make a small plasma (of the same temperature and chemical make-up) in the lab?



- Can we make a confined 250 MK plasma?
- Can we maintain it this long?
- What about all of the other reactions occurring?
- Does the plasma mimic the conditions in the star?
- How do we count the reactions?

An example. . .

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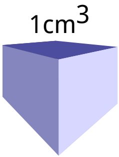

  - Typical plasma density on earth: $\rho N_A/M_H = 1 \times 10^{16}$ atoms/cm^3
  - The rate of reactions per ^{34}S $\lambda_S \approx \rho \frac{X_H}{M_H} N_A \langle \sigma v \rangle \sim 5 \times 10^{-13}$
  - So, in our “mini plasma”, $R = 0.02$ reactions per second

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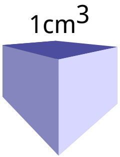

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  - So, in our “mini plasma”, $R = 0.02$ reactions per second
  - 100 reactions per hour

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- Can we maintain it this long?
- What about all of the other reactions occurring?
- Does the plasma mimic the conditions in the star?
- How do we count the reactions?

- Reaction rate (from before):

$$\langle \sigma v \rangle = \left(\frac{2\pi}{\mu k T} \right)^{3/2} \hbar^2 \sum_i \omega\gamma_i e^{-E_r/kT}$$

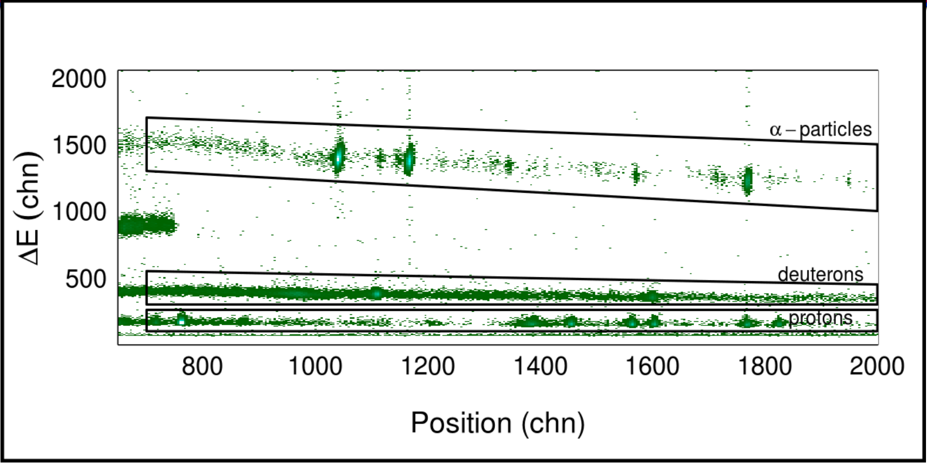
- The partial width can be calculated using

$$\Gamma_p \propto P_\ell C^2 S \theta_{sp}^2$$

- P_ℓ: Penetration factor. Depends on Coulomb force and can be calculated. *Depends strongly on E_r*
- θ_{sp}: Single-particle reduced width calculated from theory
- $C^2 S$
  - Clebsch-Gordan coefficients (calculate these)
  - Spectroscopic Factor: Must be experimentally measured. Describes how “single-particle-y” a state is.

$$\frac{d\sigma}{d\Omega} = C^2 S \frac{d\sigma}{d\Omega}_{\text{theory}}$$

Focal-plane Detector Performance - Particle ID



- Position sections function independently of ΔE
- Maximum resolution corresponds to $\Delta(\rho) \sim 0.2$ mm (2 in 10,000)
- Added wavelength-shifting fiber readout for more compact, sturdy design
- Custom-designed signal read-out electronics to improve noise characteristics

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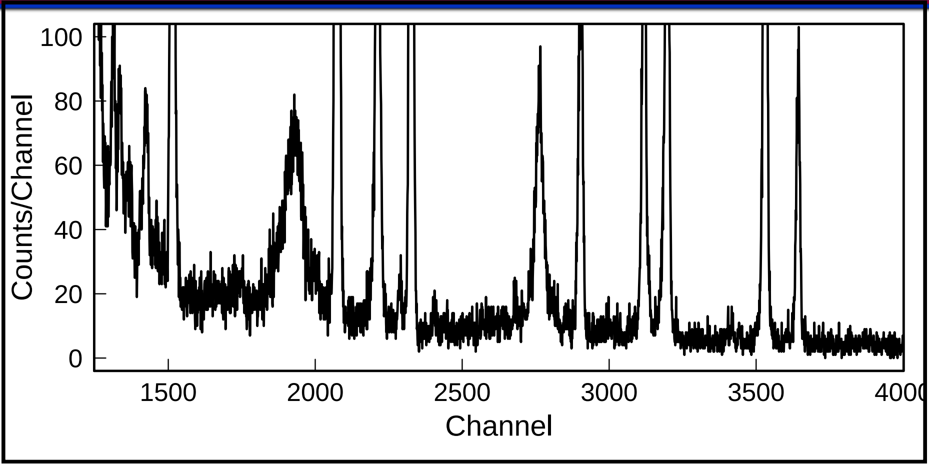

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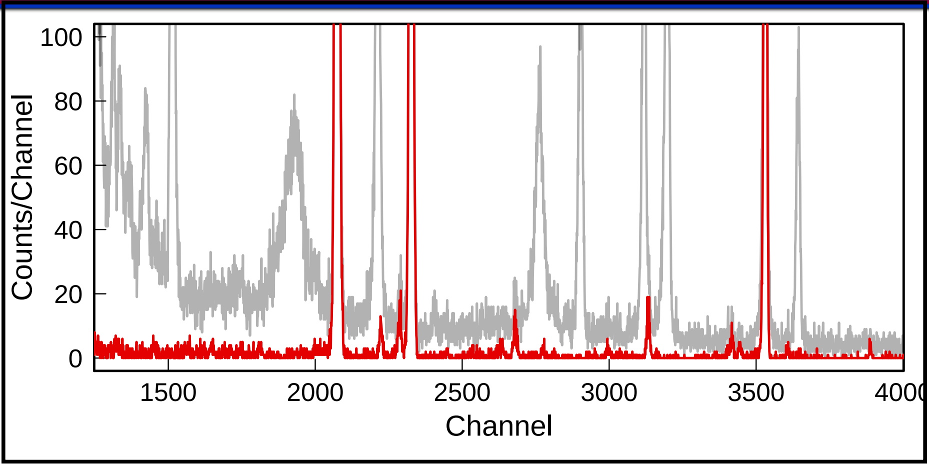

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