CONVECTIVE MIXINGS IN STELLAR EVOLUTION MODELLING

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Stellar modelling: mixing, convection and mass loss,
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ASTROPHYSICS RESEARCH INSTITUTE

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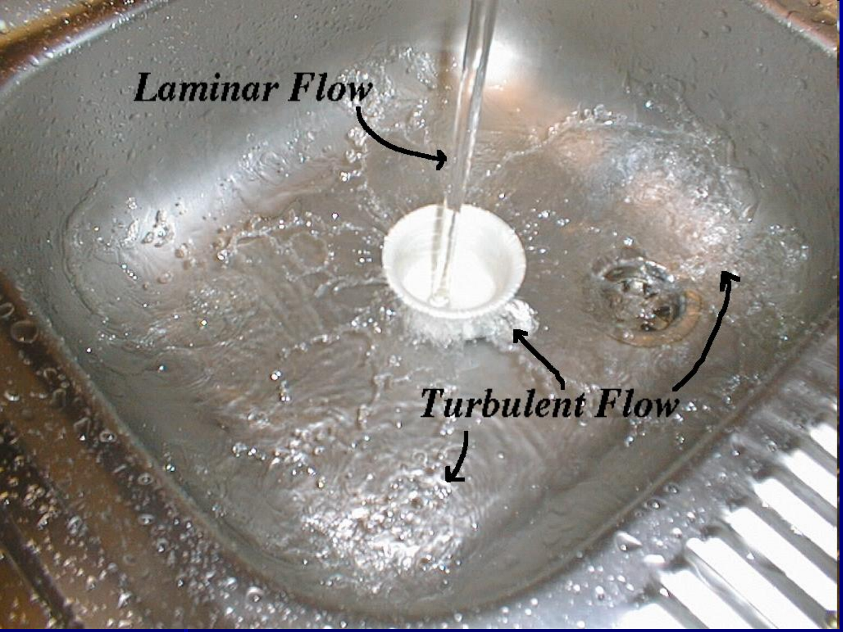
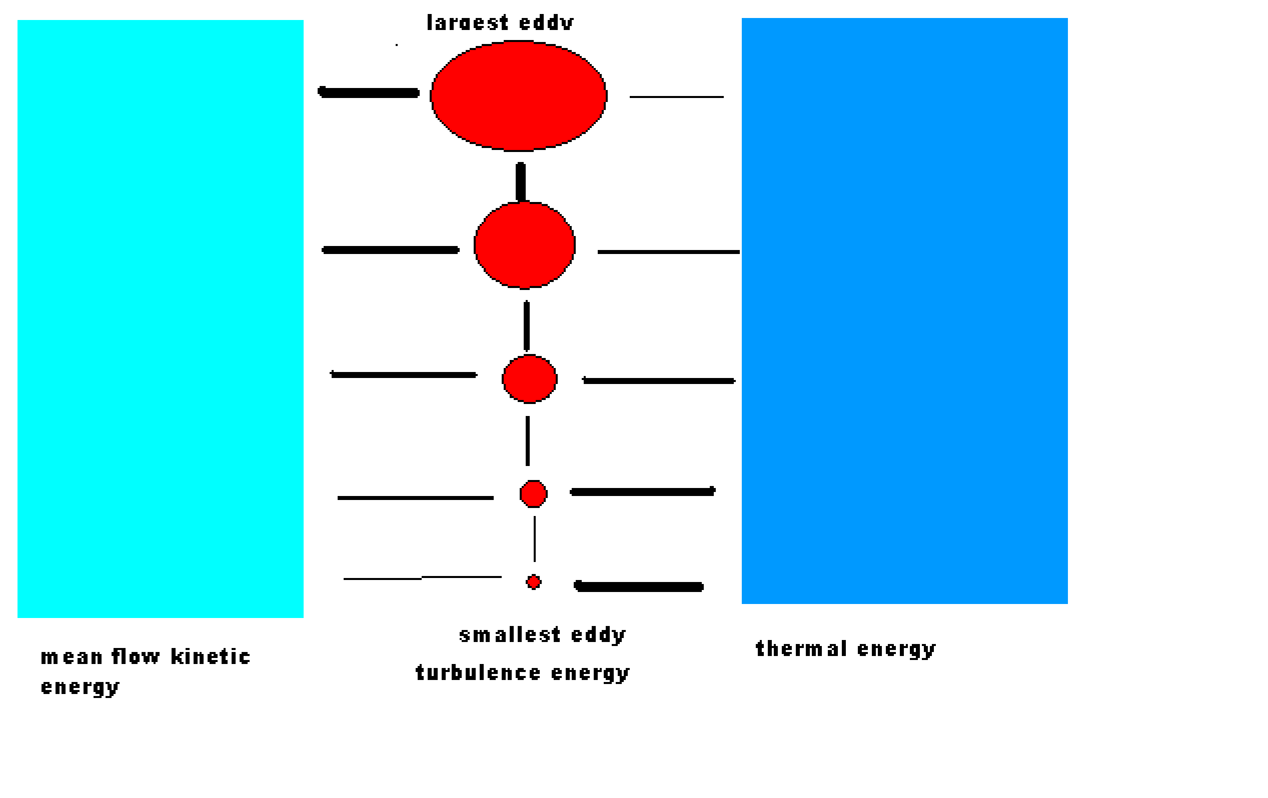


Turbulence is a flow regime characterized by chaotic and stochastic property changes. Richard Feynman described turbulence as "the most important unsolved problem of classical physics.

Re~ L/υ
length scale/viscosity

Kinetic energy transforms into turbulence energy and thence to thermal energy.
Hierarchy of turbulent eddies.
Much of kinetic energy feeds into large eddies, and is lost to heat at the smallest scales.



The treatment of convection in stellar modelling has to be able to determine:

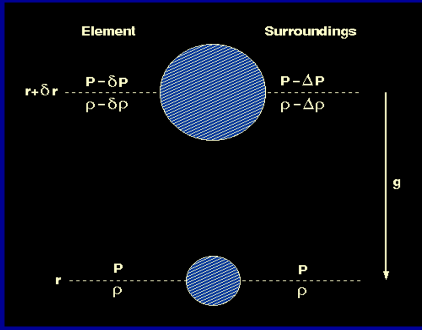
i) when a layer is stable against convection

$$\nabla_{ad} - \nabla_{rad} + \frac{\varphi}{\delta}\nabla_{\mu} \geq 0$$

$$\nabla_{ad} := \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}, \qquad \nabla_{\mu} := \frac{d \ln \mu}{d \ln P}, \qquad \nabla := \frac{d \ln T}{d \ln P},$$

$$\delta := -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{\mu,P}, \qquad \varphi := \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T}.$$

ii) whether convection can extend beyond its formal boundary;
iii) the timescale of mixing;
iv) the temperature gradient in the convective region.



Desirable (and/or necessary) features of the chosen treatment are:
Coverage of as many evolutionary phases as possible
Economy of computer time + simplicity

OUTLINE

- **Envelope convection**
- **Core convection**
- **Overshooting/semiconvection/time dependent mixing**
- **How to move forward?**

Envelope convection:
The mixing length theory (Böhm-Vitense 1958)

Almost universally used in stellar evolution codes.

Simple, local, time independent model, that assumes convective elements with mean size l, of the order of their mean free path.

	a	b	c	α
BV58	⅛	½	24	1.6-2.0
ML1	⅛	½	24	1.0
ML2	1	2	16	0.6 - 1.0
ML3	1	2	16	2.0

$l = \alpha H_p$ *mixing length*

∇' is the temperature gradient of a rising (or falling) element of matter within the convective region, ∇ is the average temperature gradient of all the matter at a given level within the convective zone (the quantity needed to solve the stellar structure equations)

$$v^2 = \frac{a l^2 g Q (\nabla - \nabla')}{H_p},$$

convective velocity

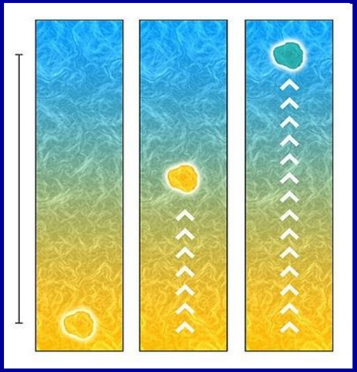
$$F_c = \frac{b \rho v C_p T l (\nabla - \nabla')}{H_p},$$

convective flux

$$\Gamma \equiv \frac{\nabla - \nabla'}{\nabla' - \nabla_{ad}} = \frac{C_p \rho^2 l v \kappa}{c \sigma T^3}.$$

convective efficiency

$$Q \equiv -(\mathrm{d}\ln\rho/\mathrm{d}\ln T)_P$$



ML2 and ML3 increase the convective efficiency compared to ML1

ORDER OF MAGNITUDE ESTIMATES WITH THE MLT

Superadiabaticity

Excess T (bubble-surroundings)

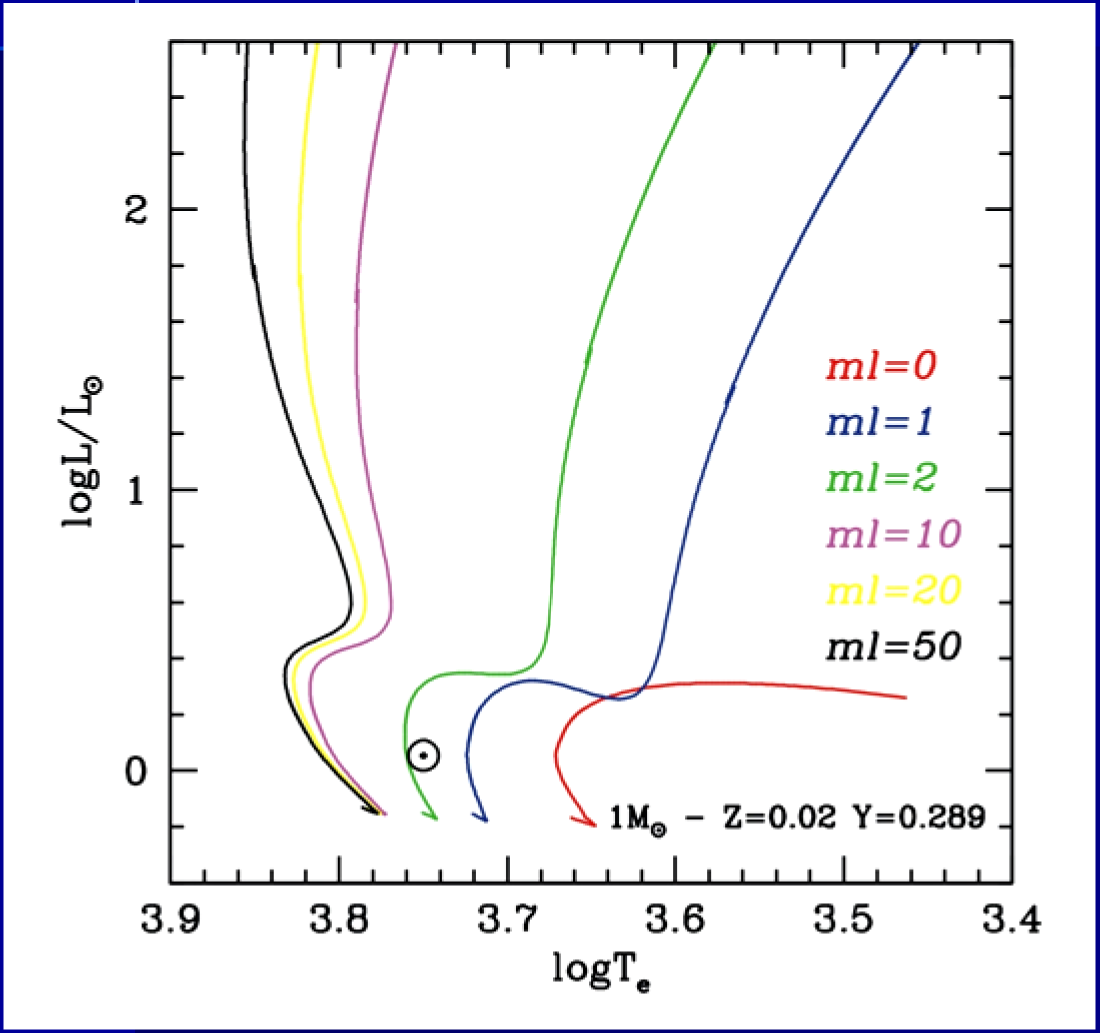
Convective efficiency

REPRESENTATIVE VALUES OF CONVECTIVE QUANTITIES IN THE STELLAR INTERIOR

Quantity	*Deep Interior*	*Outer Layers*
$\Delta\nabla T$	10^{-12} to 10^{-10} °K/cm	10^{-5} to 10^{-4} °K/cm
$\Delta T = \Lambda \Delta\nabla T$	0.01 to 1°K	100 to 1000°K
$\bar{v}/v_s$	10^{-6} to 10^{-4}	0.1 to 1
$\bar{v}$	1 to 100 cm/sec	0.5 to 15 km/sec
$t = \Lambda/\bar{v}$	1 to 100 days	1 to 100 minutes
Γ	10^{6} to 10^{9}	10^{-4} to 10^{2}

From Cox & Giuli (1968)

The value of α affects strongly the effective temperature of stars with convective envelopes

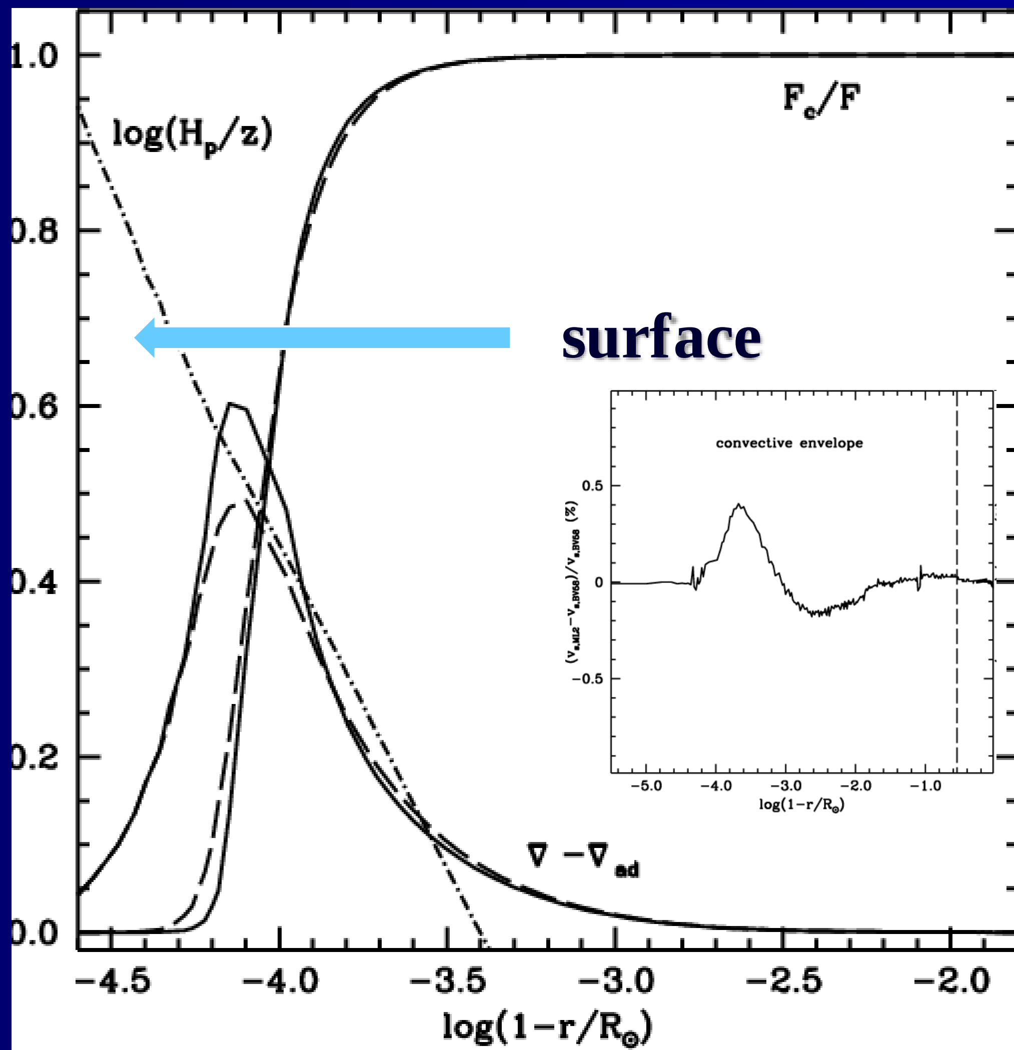


The'canonical' calibration is based on reproducing the solar radius with a theoretical solar models (Gough & Weiss 1976)

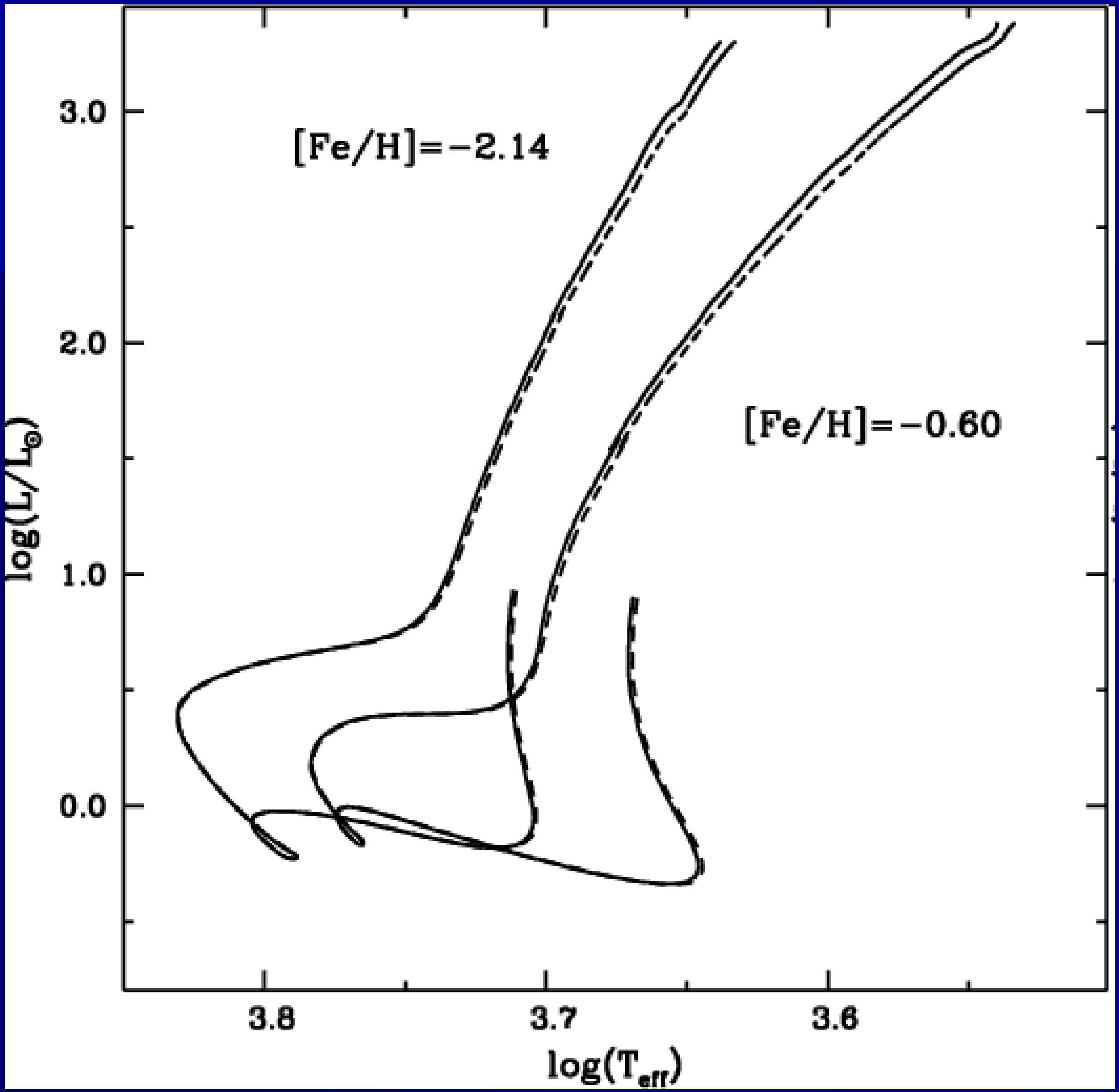
We should always keep in mind that there is a priori no reason why α should stay constant within a stellar envelope, and when considering stars of different masses and/or at different evolutionary stages

Run of the superadiabaticity $\nabla - \nabla_{ad}$ and the ratio of the convective to the total energy flux as a function of the radial location in the outer layers of the solar convection zone. Solid lines represent the ML2 model, dashed lines the BV58 model.
The dashed dotted line displays the ratio between the local pressure scale height and the geometrical distance from the top of the convective region.

BV58 α=2.01
ML2 α=0.63



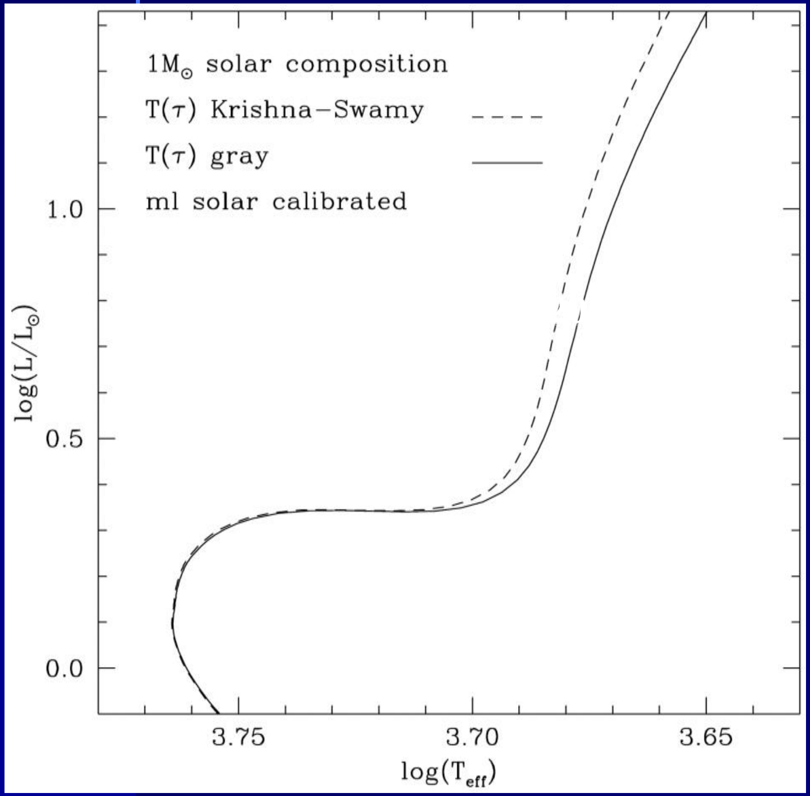
Salaris & Cassisi (2008)

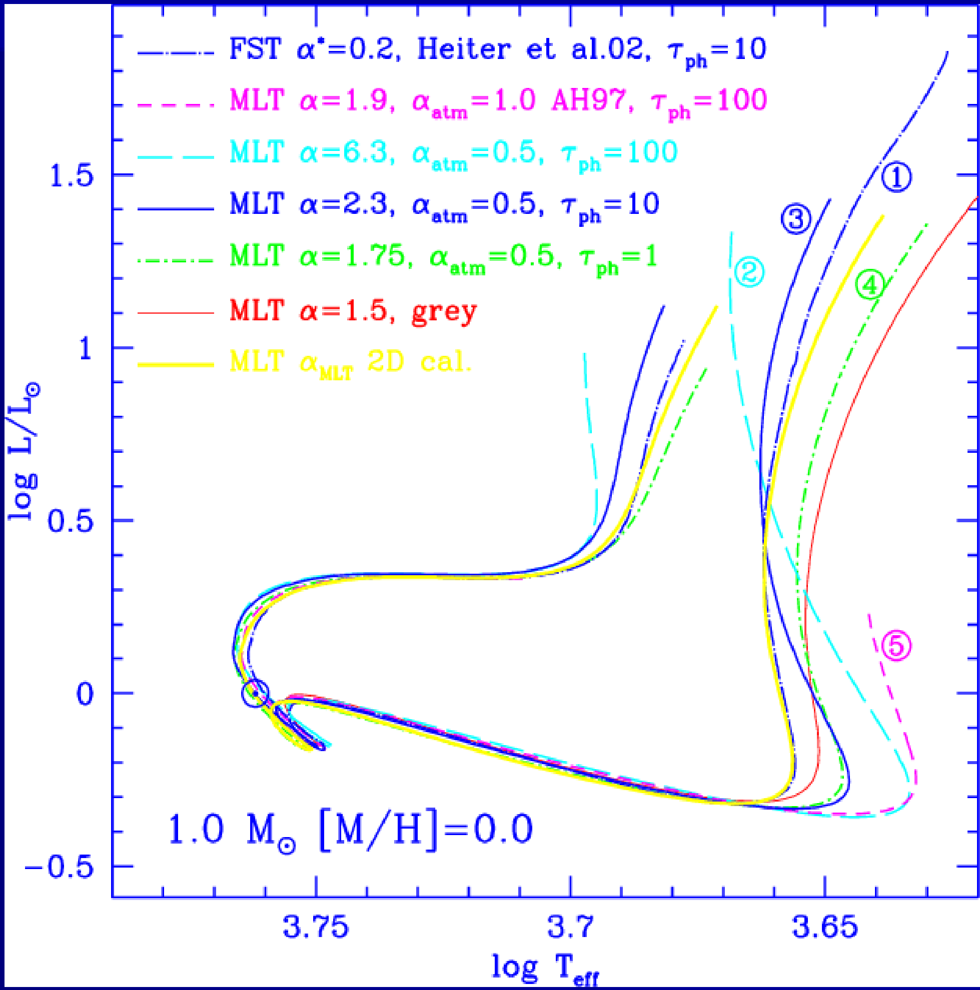


[Fe/H]=−2.14
[Fe/H]=−0.60
log(L/L$_\odot$)
log(T$_{eff}$)
3.0
2.0
1.0
0.0
3.8
3.7
3.6

Interplay between boundary condition and mixing length calibration

Boundary conditions play an important role. Solar calibrated models with different boundary conditions predict different RGB temperatures





Salaris et al. (2002)

From Montalban et al. (2004)

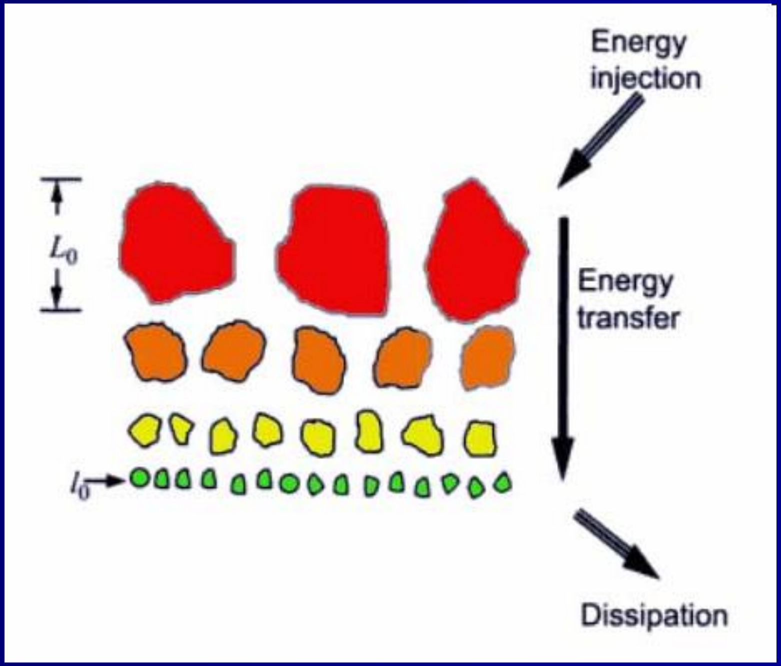
Superadiabatic convection:
The Full Spectrum of Turbulence model

The Full Spectrum of Turbulence (FST) theory (Canuto& Mazzitelli 1991, Canuto Goldman & Mazzitelli 1996) is a local theory based on an expression for the convective flux derived from a model of turbulence.

Turbulent pressure

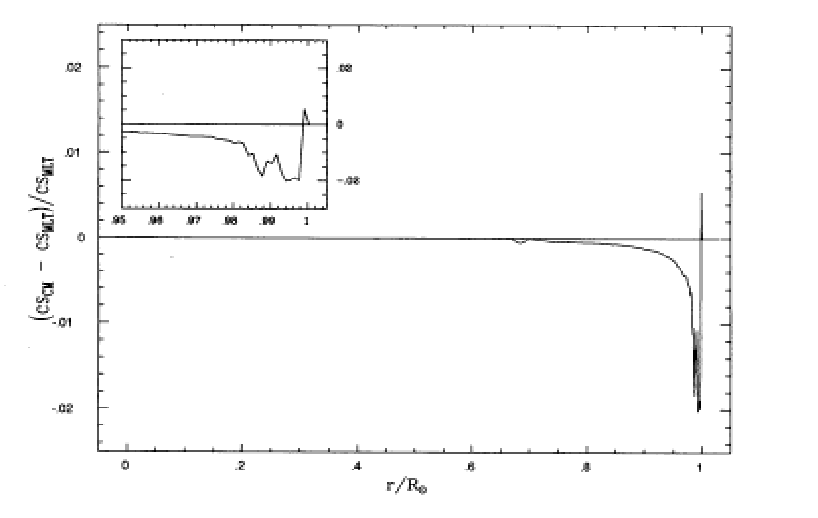
($P_{turb} \approx \rho\, v_c^2$)

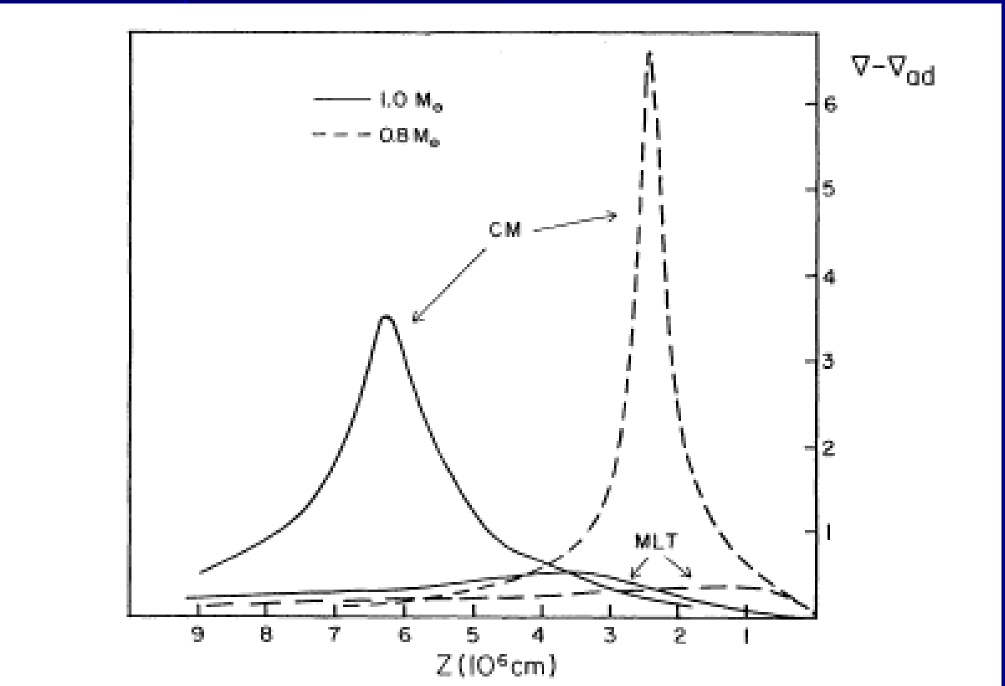
is also accounted for

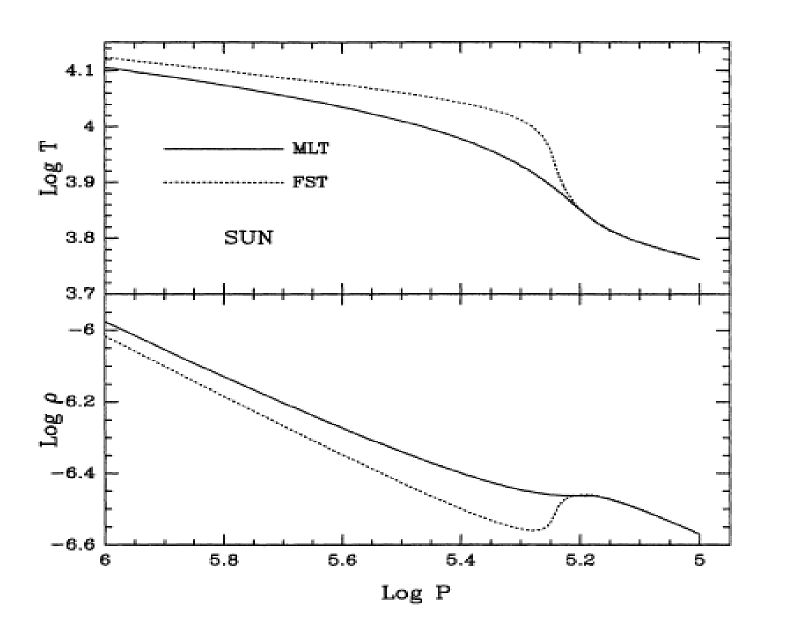


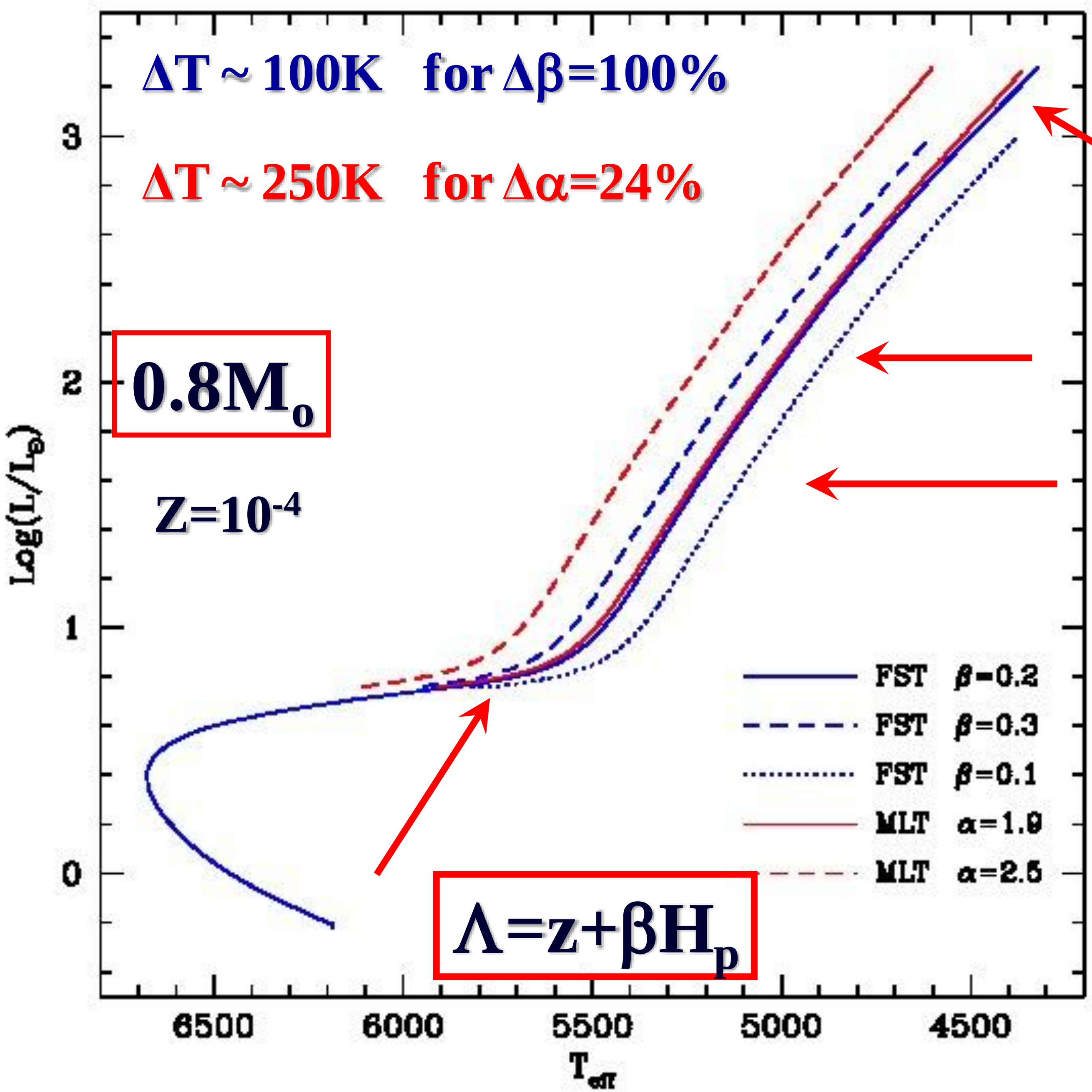
The turbulent scale length Λ is assumed to be equal to the harmonic mean of the distances from the top and bottom of the convective boundaries

F_c is about 10 times the value obtained from the MLT in realistic stellar conditions

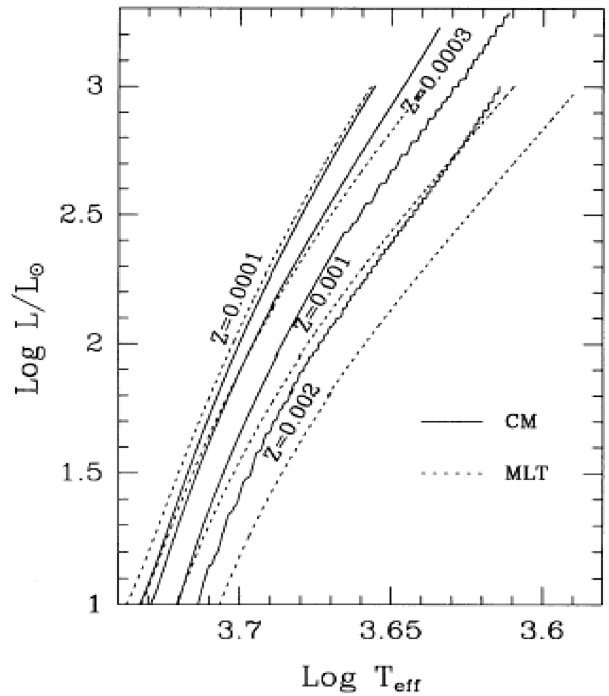


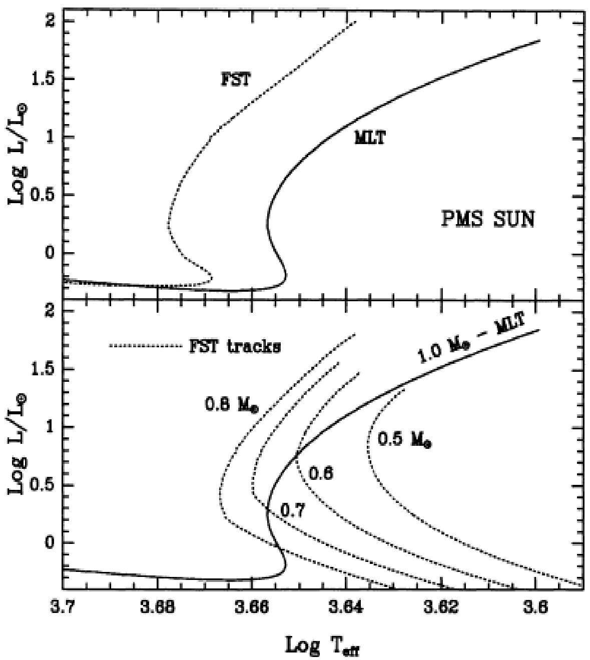


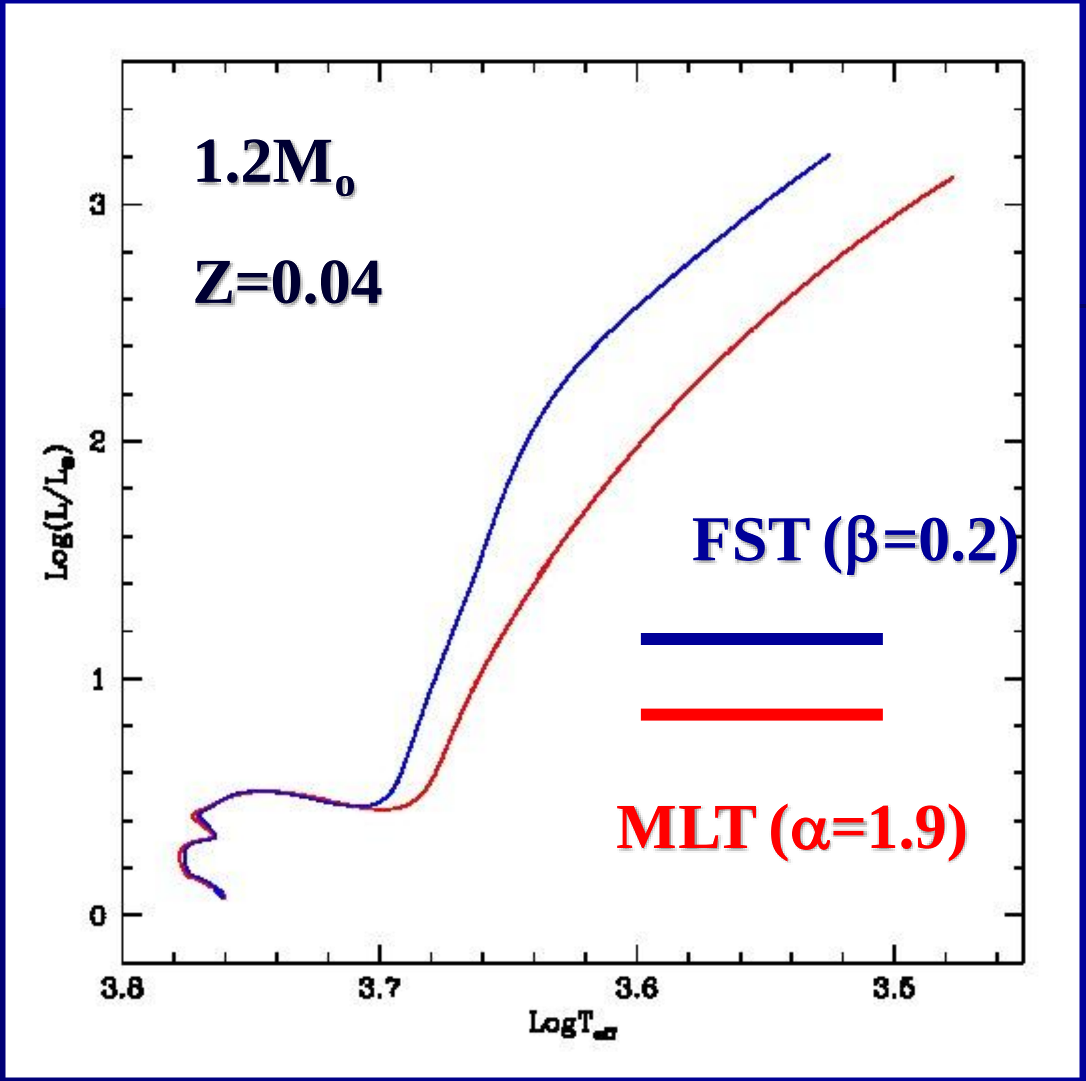




From a talk by Paolo Ventura







↑

From a talk by Paolo Ventura

Turbulent pressure $P_{turb} \approx \rho v_c^2$ is generally not included in MLT calculations. Contribution probably non-negligible in the outer layers of RGB stars

Henyey et al. (1965) have included this contribution to the pressure through an iterative procedure (P gradients coming from structure equations and H_p must contain P_{turb}, whilst thermodynamic pressure doesn't)

It is modelled in Canuto & Mazzitelli (1991) formalism

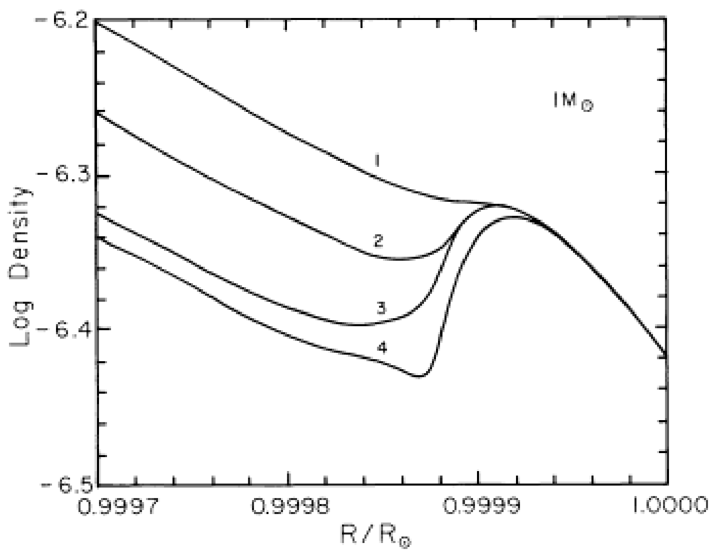


FIG. 11.—Behavior of density as a function of radius close to the surface of the Sun, for the four cases (1) MLT, $\Lambda = 1.4H_p$, no turbulent pressure; (2) new theory, $\Lambda = 0.7H_p$, no turbulent pressure; (3) new theory, $\Lambda = z$, no turbulent pressure; and (4) new theory, $\Lambda = z$, with turbulent pressure. As can be appreciated, the more complete version of the new model, curve 4, predicts a density inversion larger than any of the previous models.

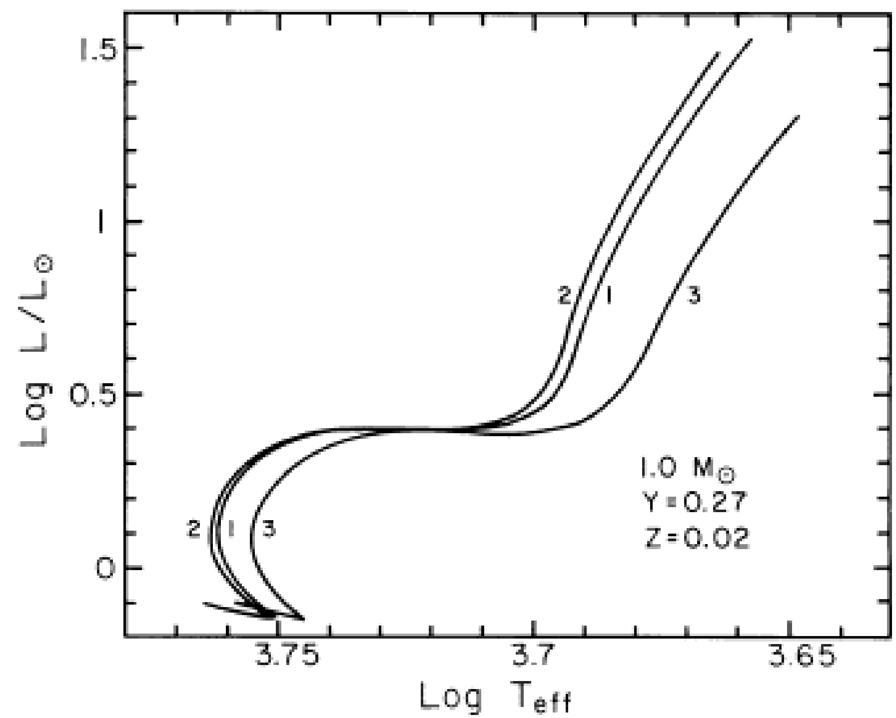
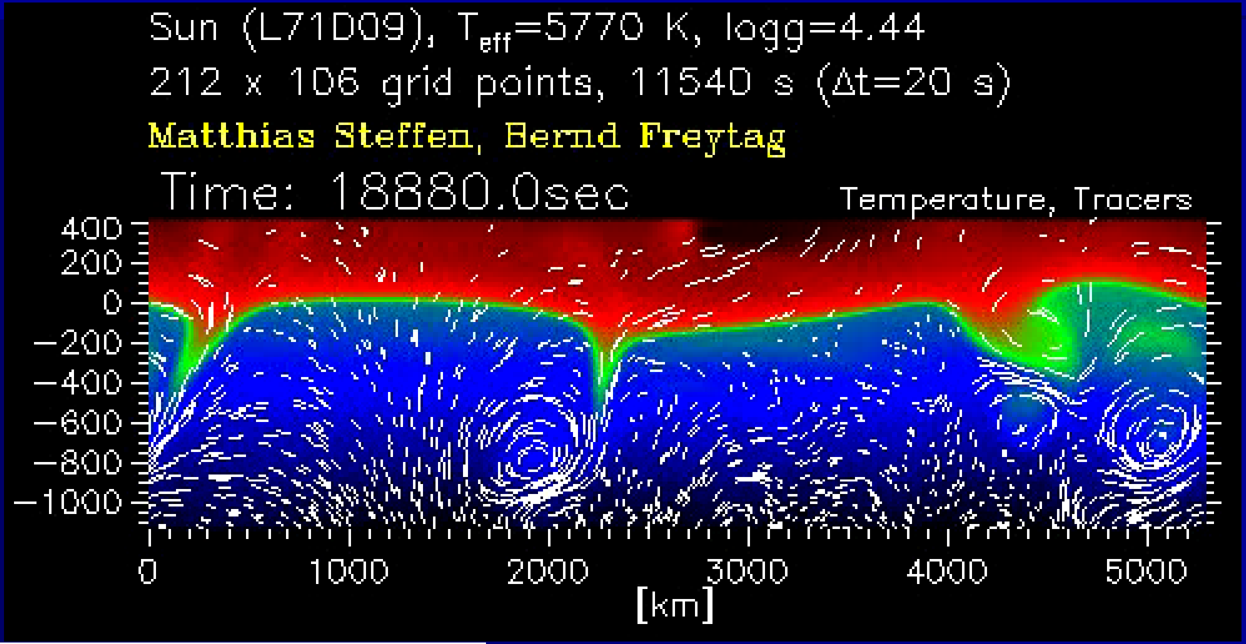
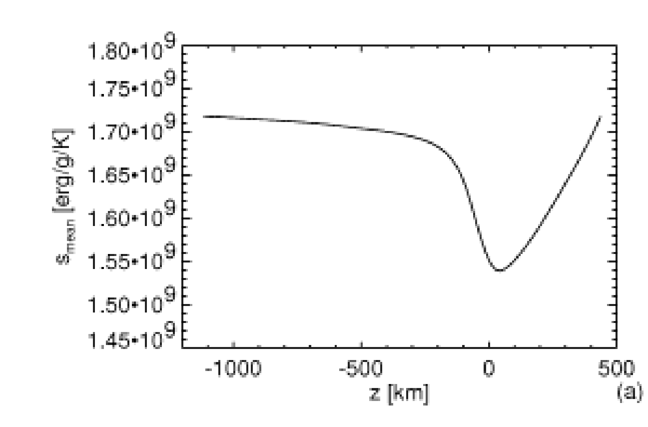


FIG. 10.—Evolutionary tracks in the H-R diagram for a 1 $M_\odot$ star, with $Y = 0.27$ and $Z = 0.02$, for (1) new theory, $\Lambda = z$, with turbulent pressure; (2) new theory, $\Lambda = z$, no turbulent pressure; and (3) MLT, $\Lambda = z$, no turbulent pressure.

2D hydro-simulations

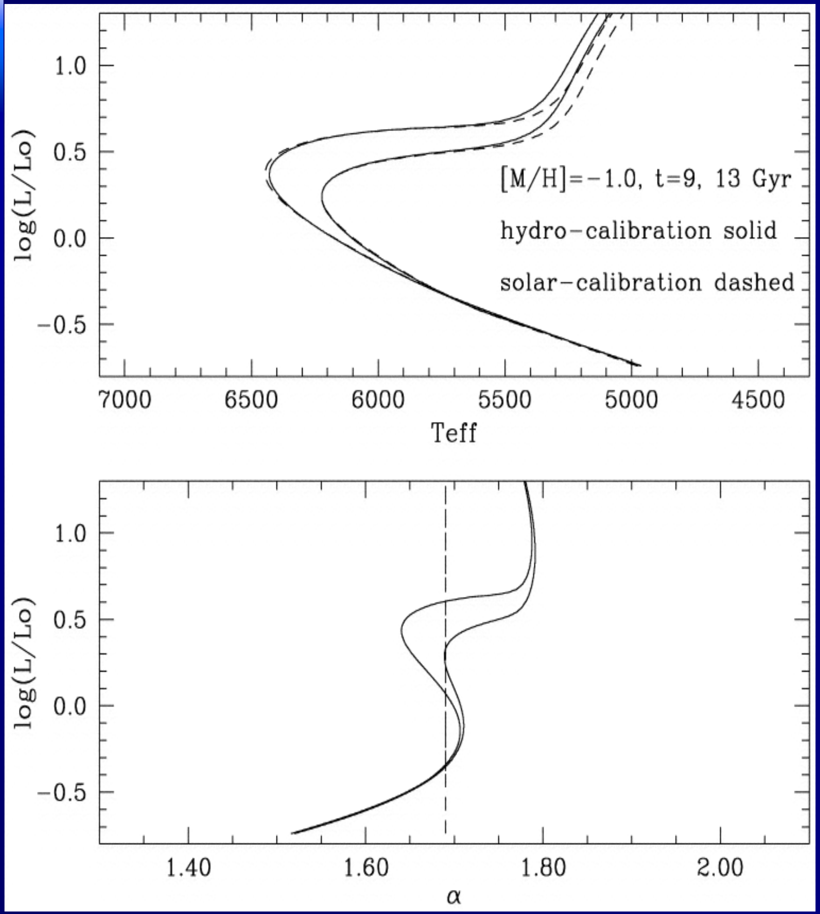




Ludwig et al. (1999)

Hydro-calibration

Previous attempts by Deupree & Varner (1980) Lydon et al (1992, 1993)



From Freytag & Salaris (1999)

Extended grid of 2D hydro-models by Ludwig, Steffen & Freytag

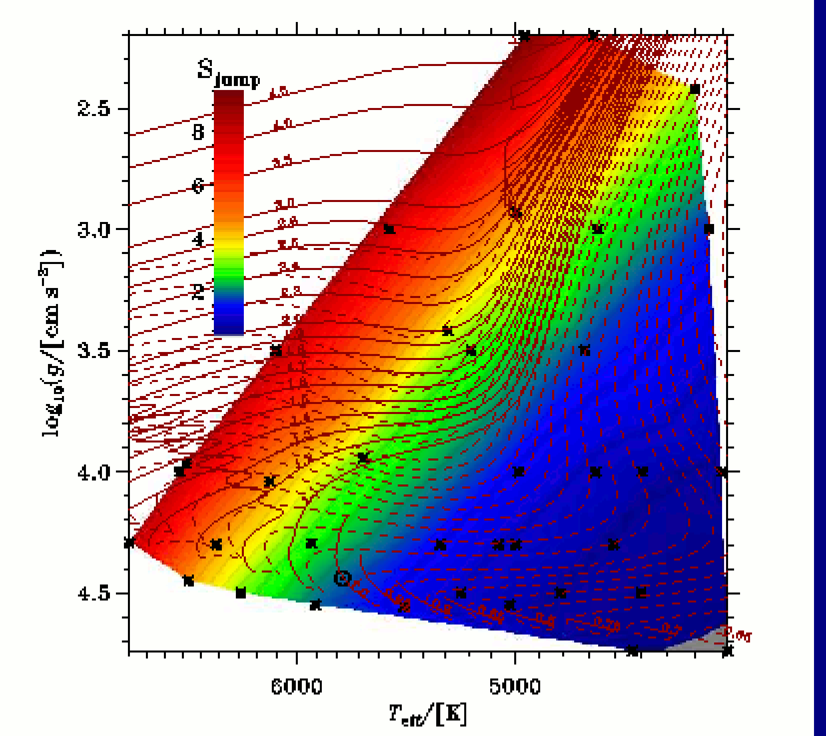
Static envelope models based on the mixing length theory calibrate α by reproducing the entropy of the adiabatic layers below the superadiabatic region from the hydro-models.

A relationship $\alpha = f(T_{eff}, g)$ is produced, to be employed in stellar evolution modelling

(Ludwig et al.1999)

3D hydro-simulations

Trampedach et al. (2013)



the atmospheric entropy jump $S_{jump}/[10^8 \mathrm{erg\,g^{-1}\,K^{-1}}]$, a measure of convective efficiency, as function of stellar atmospheric parameters.

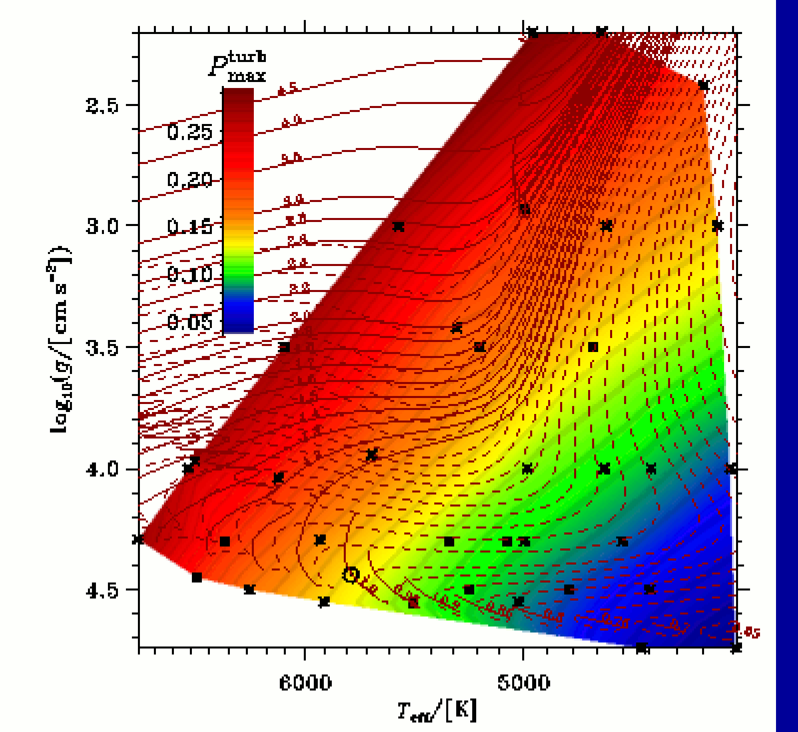


Figure 6. As Fig. 1, but showing the maximum (with depth) of the $P_{turb}/(P_{turb} + P_{gas})$-ratio, as function of stellar atmospheric parameters.

Another hydro-motivated calibration Arnett et al. (2010)

MLT Choice	a	b	c	ℓ
BV58	0.125	0.5	24	Free parameter
ML2	1	2	16	Free parameter
AMY	≈0.1	0.256	$24/g_{ML}$	$\min(4H_P, \ell_{CZ})$

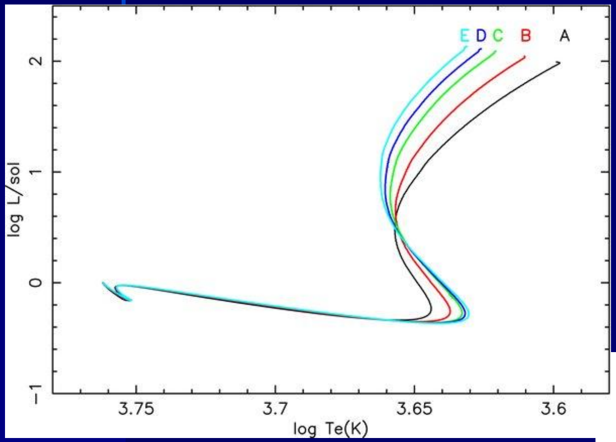
g_{ML} needs to be calibrated from hydro-simulations of the superadiabatic regions

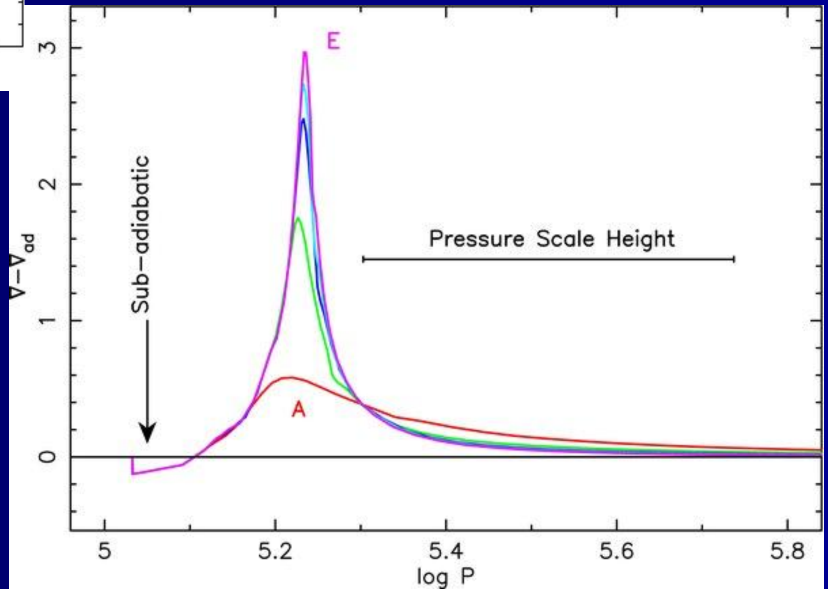
$g_{ML} = (\ell/\sqrt{3}r_b)^2$, where r_b is the radius of a blob just contained inside the SAR.

Table 2
Solar Models with MLT

Model	α_{ML}	g_{ML}	$R/R_\odot$	$L/L_\odot$	$r_{CZ}/R_\odot$	He_{surf}	v_m (km s^{-1})
A	1.650	1.0	1.001	1.000	0.7169	0.2379	2.25
B	2.323	42.0	1.001	1.000	0.7172	0.2378	2.80
C	3.286	270.0	1.001	0.9997	0.7173	0.2377	3.05
D	4.000	595.0	1.000	0.9997	0.7168	0.2373	3.20
E	5.190	1540.0	1.001	0.9998	0.7172	0.2377	3.40
Sun	...	...	1.000	1.000	0.713 ± 0.001	0.24	3.20[a]

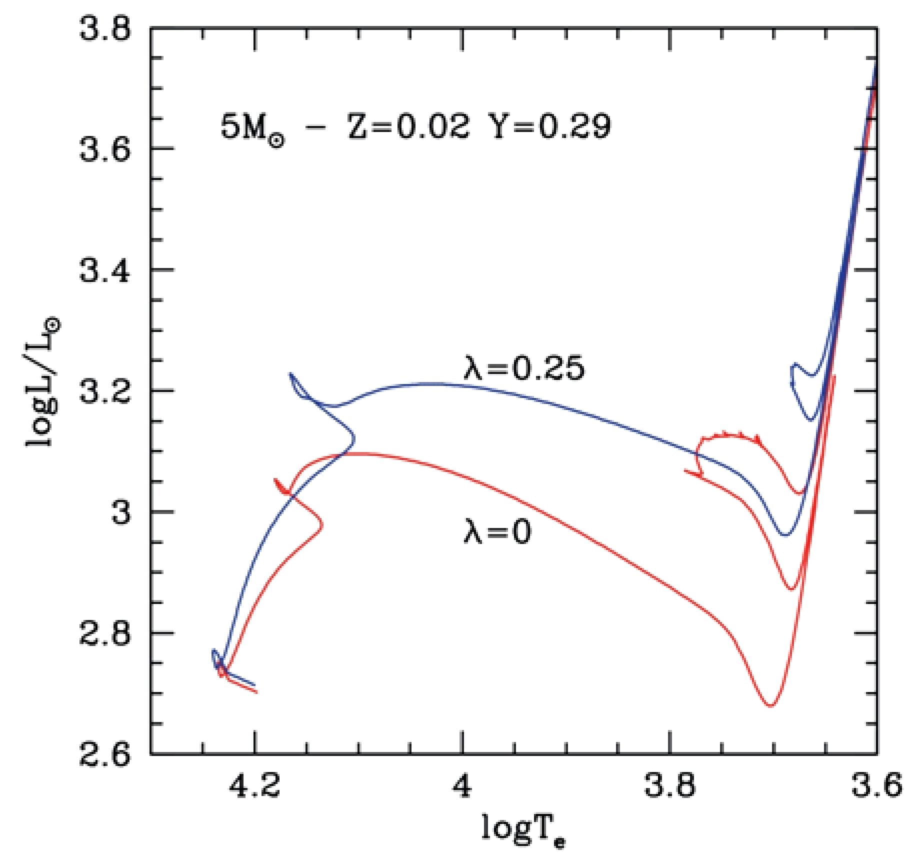
Note. [a] Inferred from the model data in Asplund et al. (2005).





Core Convection (overshooting)

Extension of the mixed region beyond the formal convective boundary



The H central burning lifetime is longer;

The size of the He core at the end of the central H-burning phase is larger; The mean luminosity during the central He-burning phase is larger.

The central He-burning lifetime is shorter;

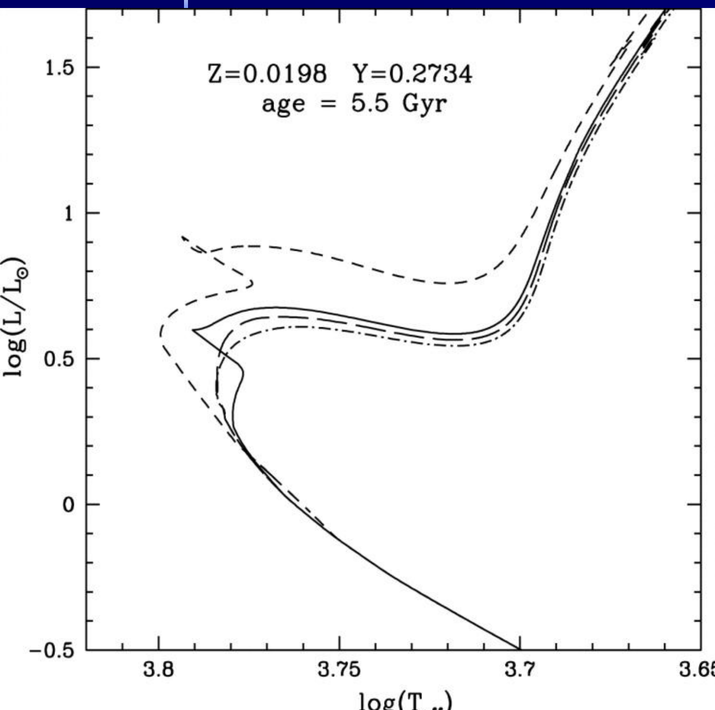
The critical masses M_{HeF} and M_{up} are significantly reduced

Empirical calibrations tell us whether mixing beyond the formal convective boundary is necessary, but this may not be entirely due to an extended convective mixing

Overshooting extension as a function of stellar mass

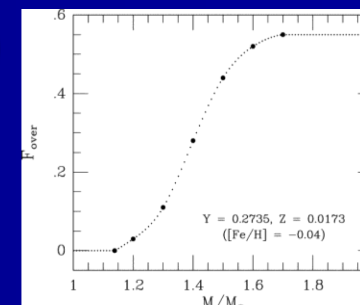
i) **Y^2 and BaSTI**

λ_{ov} decreasing with decreasing stellar mass below a given threshold, and constant λ_{ov} (typically ~ 0.2 Hp) for larger masses.



ii) **Vandenberg et al. (2006)**

$$\int_0^{r_0} F_{\rm over}(L_{\rm rad}-L)\frac{1}{T^2}\frac{dT}{dr}\,dr + \int_{r_0}^{r_{cc}} (2-F_{\rm over})(L_{\rm rad}-L)\frac{1}{T^2}\frac{dT}{dr}\,dr = 0. \qquad (2)$$



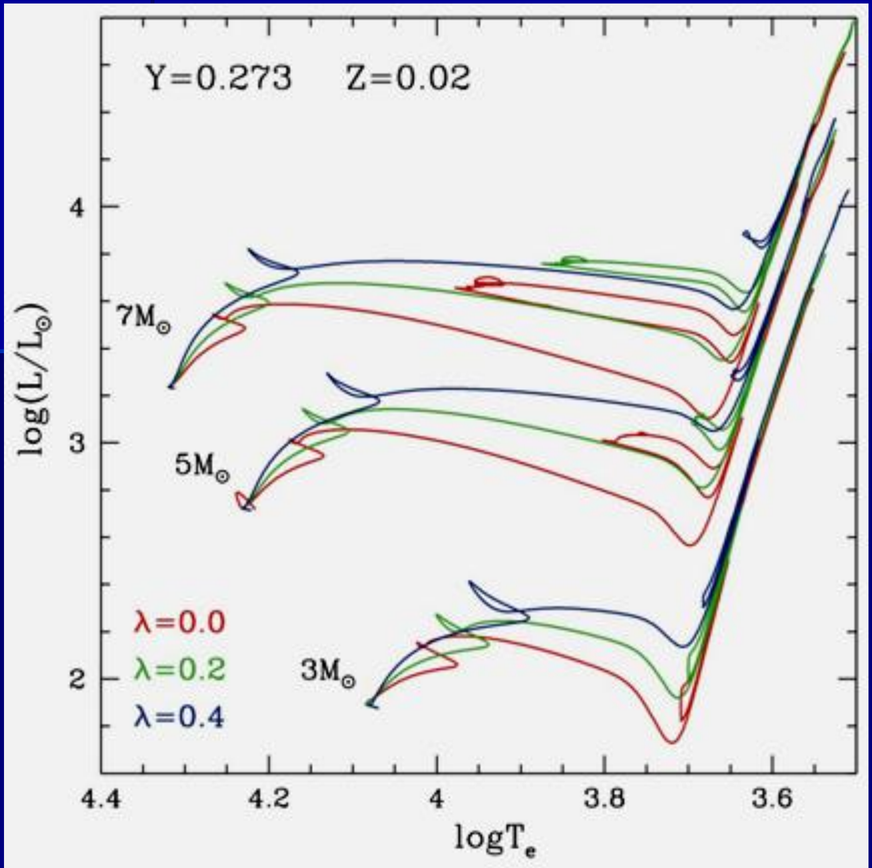
iii) **Pols et al. (1997) parametrize the extension of the overshooting region in terms of a stability criterion**

$$\nabla_r > \nabla_a - \delta$$

$$\delta = \delta_{ov} / (2.5 + 20\zeta + 16\zeta^2)$$

ζ = P_{rad}/P_{gas}

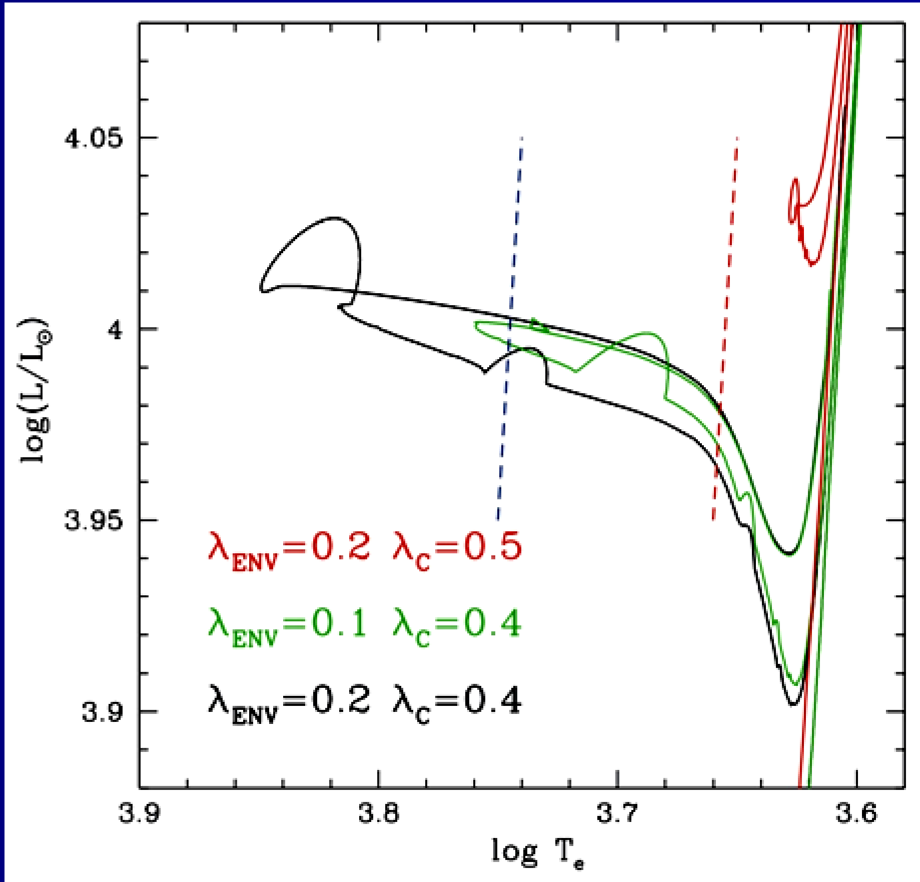
δ_{ov} =0.12



from Cassisi (2004)

Core overshooting on the MS tends to suppress blue loops

7 Mo



Envelope overshooting tends to restore blue loops

see also Alongi et al. (1992)

Time dependent mixing/overshooting

See also, i.e., Eggleton (1972), Pols & Tout (2001), Langer et al. (1983), Ventura & Castellani (2005)

Herwig et al. (1997)

$$\frac{dX_i}{dt} = \left(\frac{\partial X_i}{\partial t}\right)_{\rm nuc} + \frac{\partial}{\partial M_r}\left[\left(4\pi r^2 \rho\right)^2 D \frac{\partial X_i}{\partial M_r}\right] \qquad (1)$$

$D_r = 0$ Radiative region

$D_c = 1/3 v_c l$ Convective region

$$D_{os} = D_0 \exp\frac{-2z}{H_v}, \quad D_0 = v_0 \cdot H_p, \quad H_v = f \cdot H_p, \qquad (3)$$

Overshooting

f=0.02

v_c , v_o from MLT

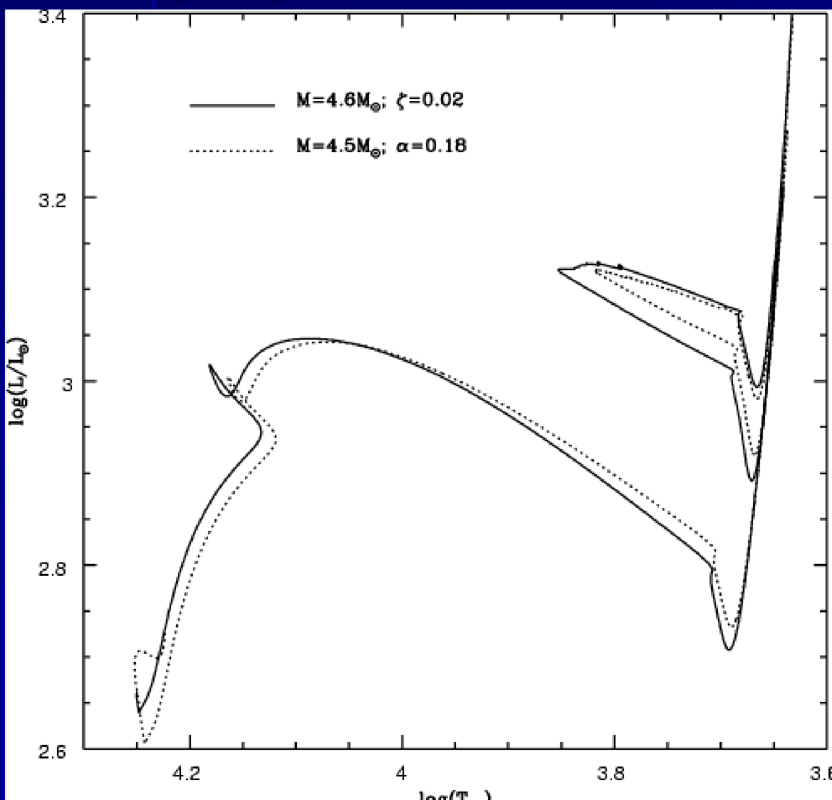
Following Freytag et al. (1996) hydro-simulations

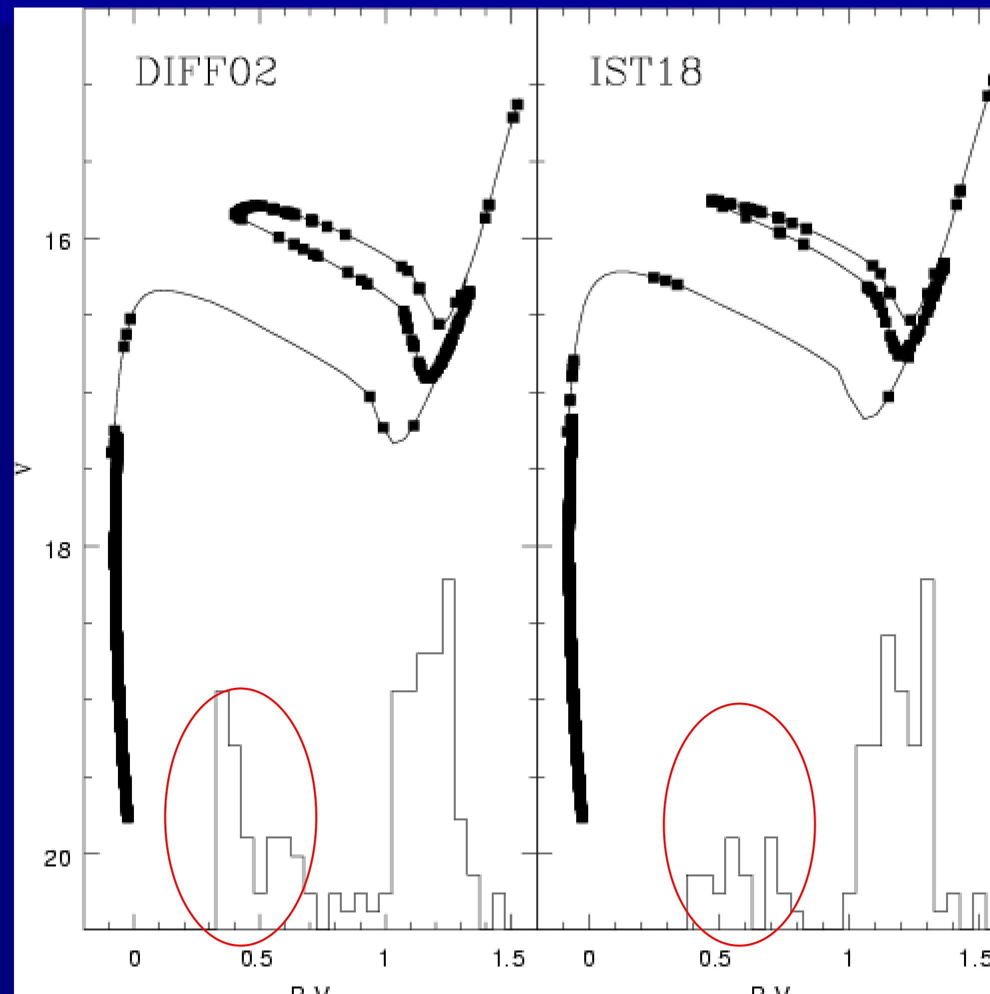
Time dependent mixing in convective cores

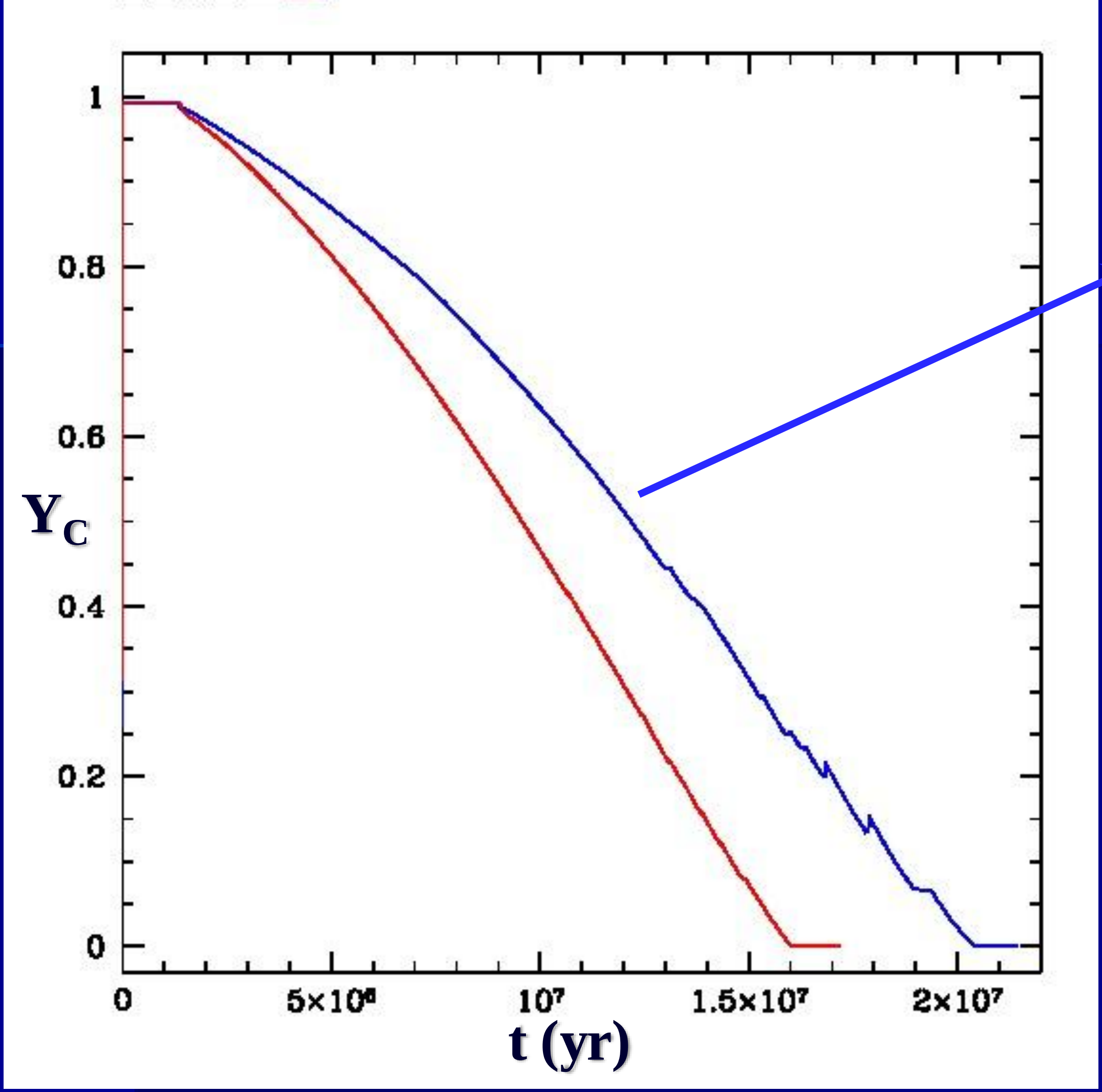
Ventura & Castellani (2005 – but see also, e.g., Eggleton 1972, Deng et al. 1996, Salasnich et al. 1999, Woosley et al. 2002)

150 Myr Z=0.008

Approach similar to Herwig et al. (1997)







In the diffusive case helium is consumed more slowly in the core

From a talk by Paolo Ventura

Semiconvection

$$\nabla_{ad} - \nabla_{rad} + \frac{\varphi}{\delta}\nabla_{\mu} \geq 0$$

Stability according to Ledoux criterion

$$\nabla_{ad} := \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad}, \qquad \nabla_{\mu} := \frac{d \ln \mu}{d \ln P}, \qquad \nabla := \frac{d \ln T}{d \ln P},$$

$$\delta := -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{\mu,P}, \qquad \varphi := \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T}.$$

Semiconvection is a secular instability which can occur in non-rotating stars. According to a stability analysis by Kato (1966), it is an oscillatory instability which appears in regions where an unstable temperature gradient is stabilized against convection by a sufficiently large gradient in the mean molecular weight (μ-gradient), i.e., it lives in the regime

$$\frac{\varphi}{\delta}\nabla_{\mu} \geq \nabla_{rad} - \nabla_{ad} \geq 0$$

(Kippenhahn & Weigert 1991; and Fig. 2.4). Heat transfer between a displaced mass element and its surrounding causes the growth of the instability on the local thermal time-scale.

Diffusion treatment

In STERN, semiconvection is treated following Langer, Sugimoto & Fricke (1983). The diffusion coefficient for this process is computed from

$$D_{\rm sem} = \frac{\alpha_{\rm sem} K}{6 c_{\rm P} \rho} \frac{\nabla - \nabla_{\rm ad}}{\nabla_{\rm ad} - \nabla + \frac{\varphi}{\delta}\nabla_\mu} , \qquad K = \frac{4acT^3}{3\kappa\rho} ,$$

As proposed by Langer (1991a), an efficiency parameter of $\alpha_{\rm sem} = 0.04$ is adopted

In KEPLER semiconvection is computed from

$$D'_{\rm sem} = \frac{1}{6} {\alpha_{\rm MLT}}^2 v_{\rm sem} H_{\rm P} ,$$

$v_{\rm sem}$ is determined through

$$v_{\rm sem} = \sqrt{(\nabla - \nabla_{\rm ad}) \frac{P\delta}{g\rho^2} \frac{{\rm d}P}{{\rm d}r}} .$$

The diffusion coefficient is limited to a fraction $\alpha_{\rm sem}$ of the radiative diffusion coefficient

$$D_{\rm rad} = \frac{K}{\rho c_V}$$

by means of

$$D_{\rm sem} = \frac{\alpha_{\rm sem} D_{\rm rad} D'_{\rm sem}}{D'_{\rm sem} + \alpha_{\rm sem} D_{\rm rad}} .$$

$$\alpha_{\rm sem} = 10^{-4}$$

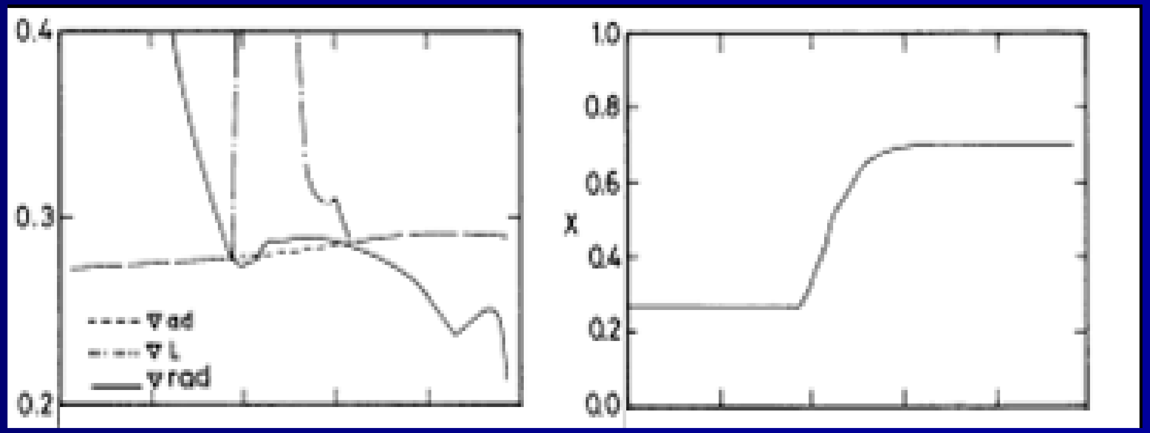
Semiconvection in massive stars

Convective mixing outside the central H-burning core following the end central H-depletion.

See, e.g. Schwarzschild & Härm (1958), Chiosi & Summa (1970), Stothers (1970), Simpson (1971), Eggleton (1972), Langer et al. (1983)

$$\nabla_L = \nabla_{ad} + \frac{\varphi}{\delta}\nabla_\mu$$

30 M_o



M_r/M M_r/M

The presence of a fully mixed shell favours a blue He ignition

Decreasing efficiency of mixing → He ignition increasingly to the red

The B/R ratio of supergiants can in principle be used to calibrate the efficiency of mixing in semiconvective regions (Langer & Maeder 1995)

Horizontal Branch semiconvection with instantaneous mixing

I phase : Core Expansion

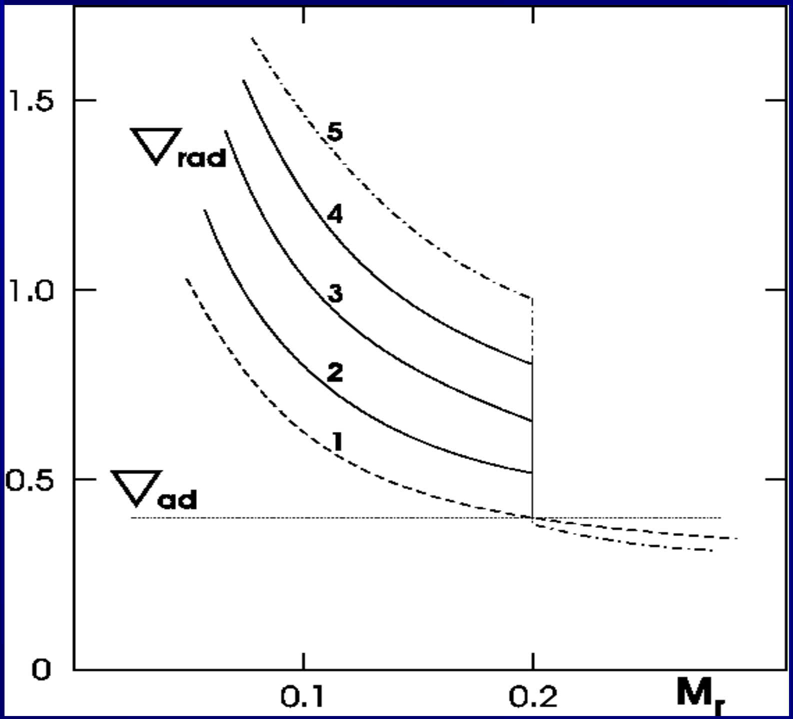
Mass of fully mixed core increases

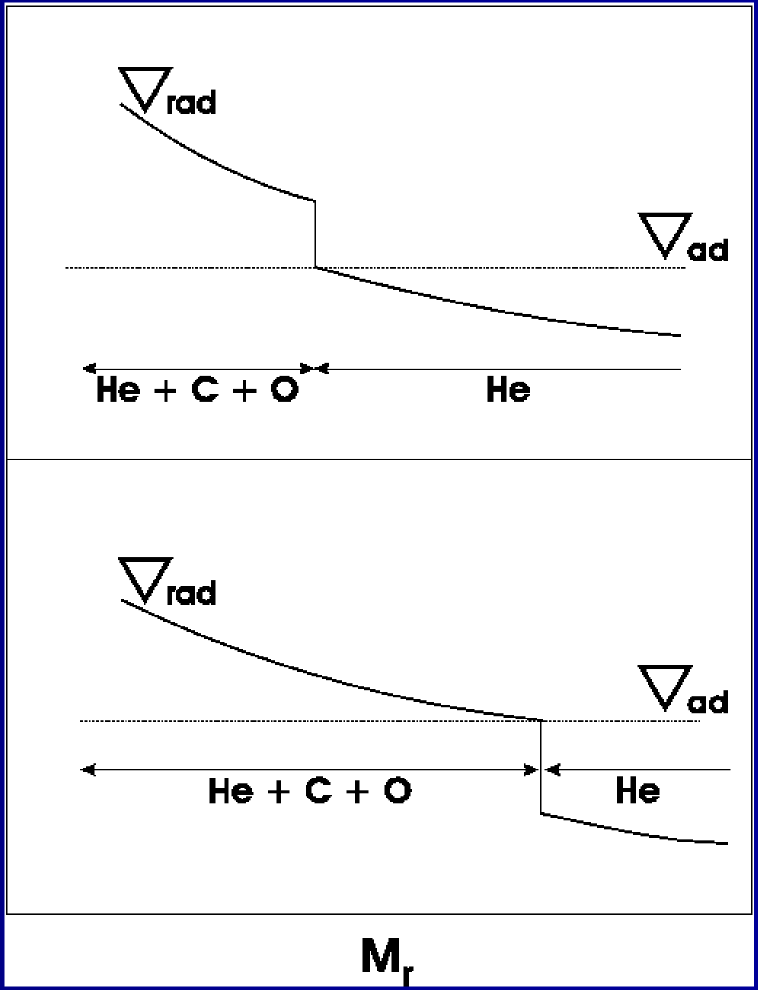
C produced by He-burning

Opacity increases $F_c \approx \nabla_{rad} - \nabla_{ad}$

Radiative gradient discontinuity at

the convective core boundary

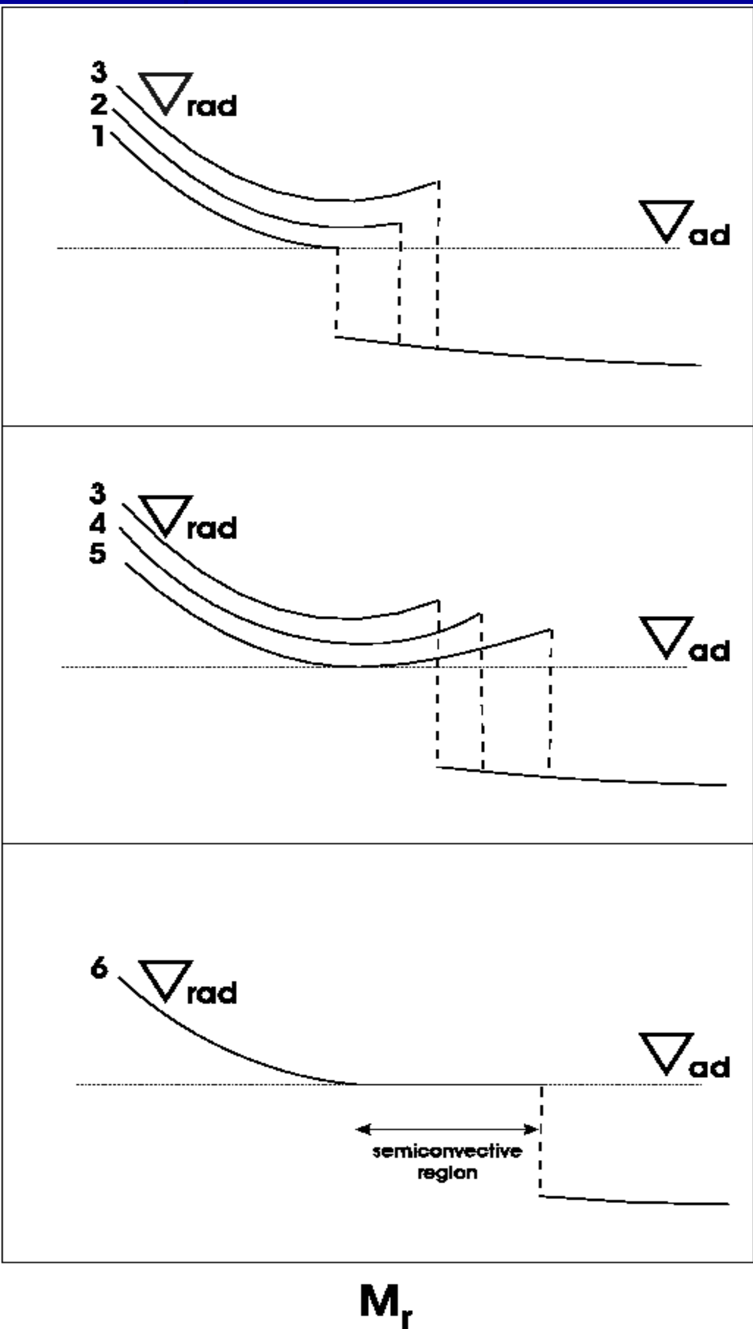


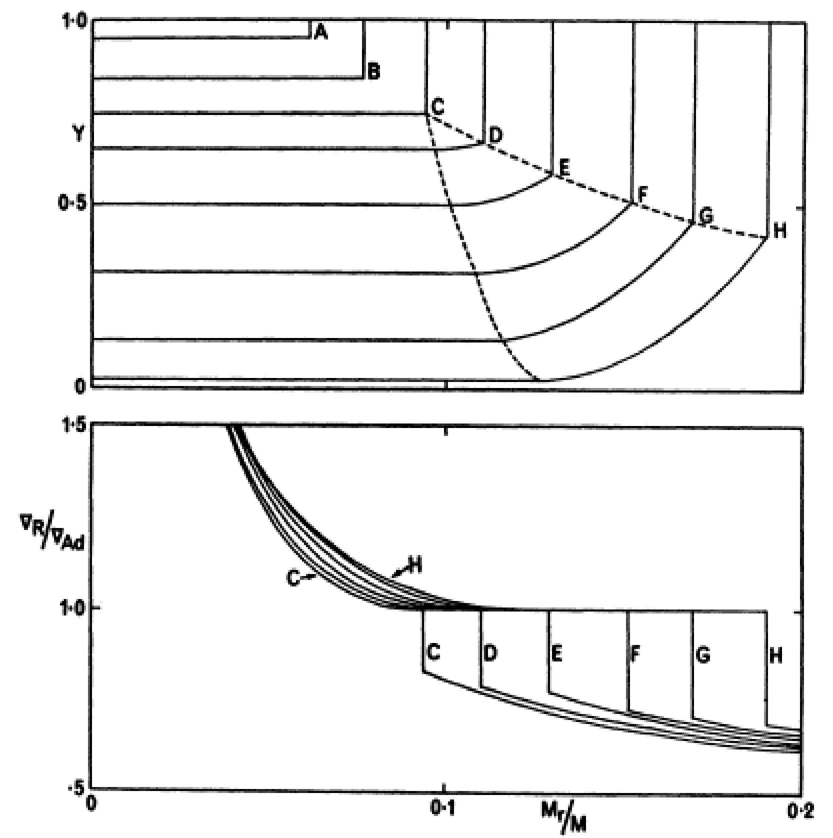


See, e.g. Castellani et al. (1971)

II phase: Development of a 'partial mixing zone'

When Y_c decreases below ~0.7, a 'partial mixing' (semiconvective) zone develops beyond the boundary of the convective core.

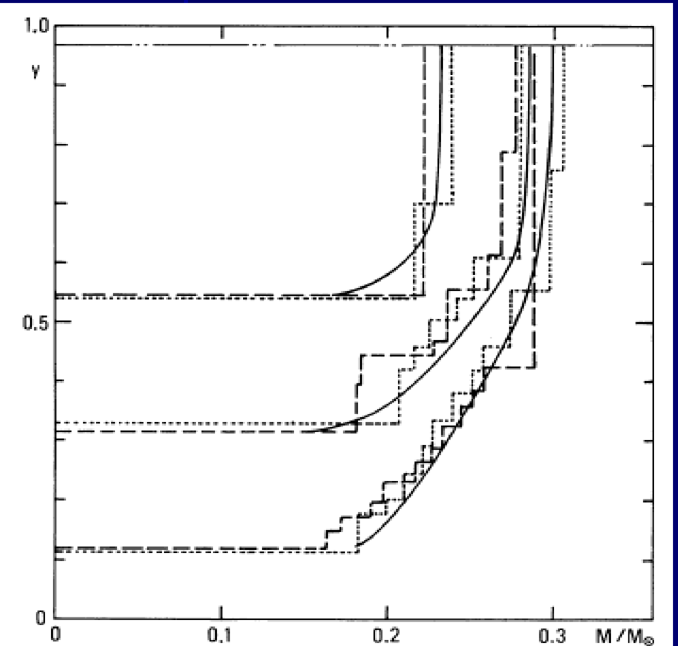




Mimicking semiconvection with overshooting

Caloi & Mazzitelli (1990)

Extension of mixing (by ~0.1Hp) in regions beyond the boundary of all convective regions forming within the He-rich core

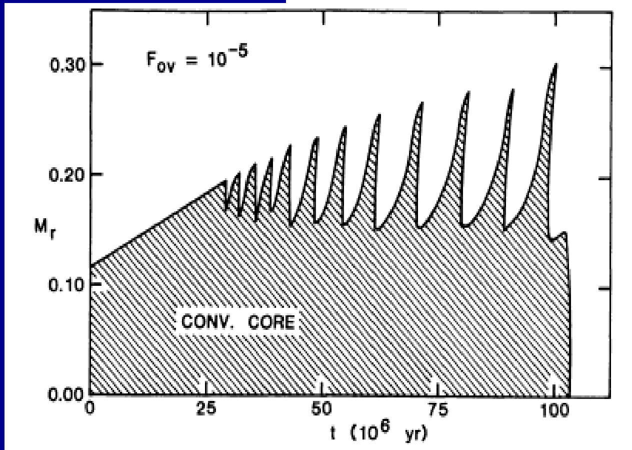


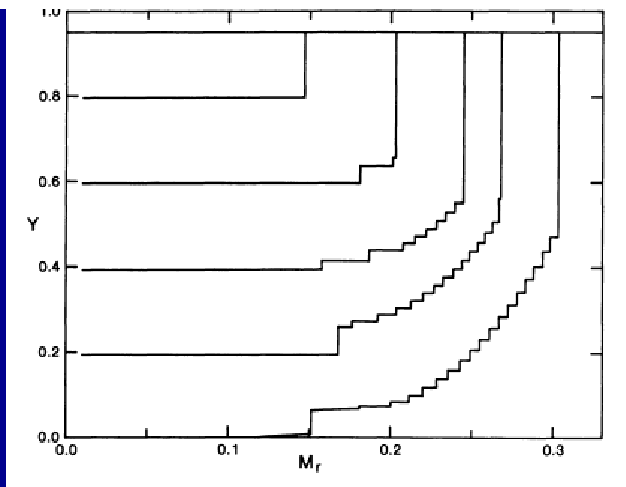
The chemical profiles of helium inside the core at three different evolutionary phases for the cases with semi-convection (solid lines), overshooting of 0.05 Hp (dashed lines) and 0.1 Hp (dotted lines)

Sweigart (1990)

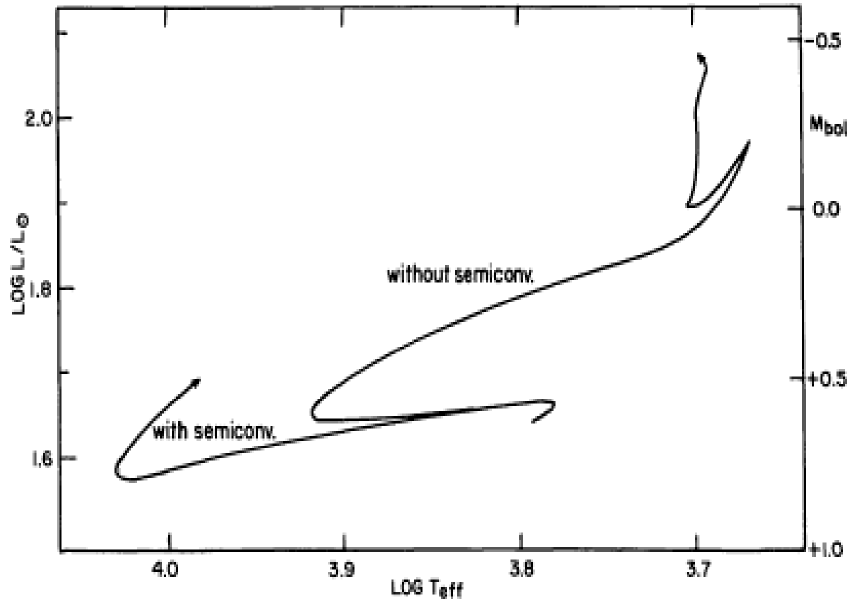
The edge of the convective core is let propagate with velocity

$$v_c = F_{ov} \frac{\nabla_{rad} - \nabla_{ad}}{\mu_i - \mu_e}$$





Breathing pulses are still found to occur



‹‹‹‹‹‹‹‹ Semiconvection and HRD evolution

Semiconvection increases central He-burning lifetime by a factor ~1.5 - 2

Breathing Pulses ››››››››

(Start when Y_c~0.10)

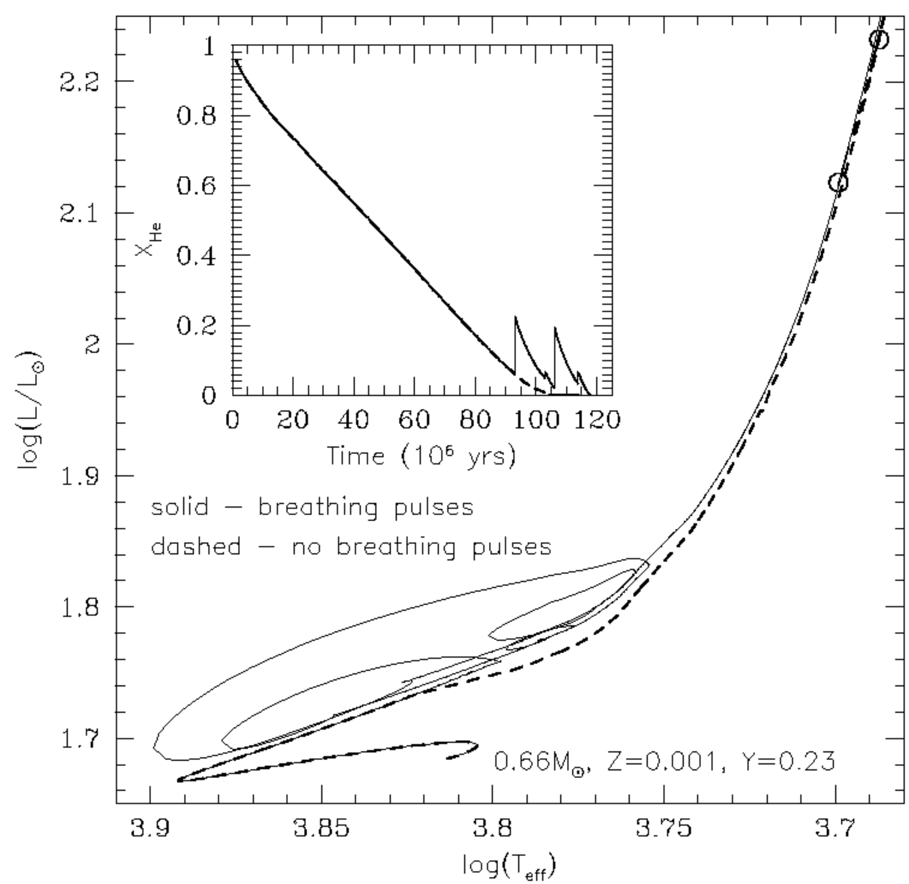
Numerical artifact ??

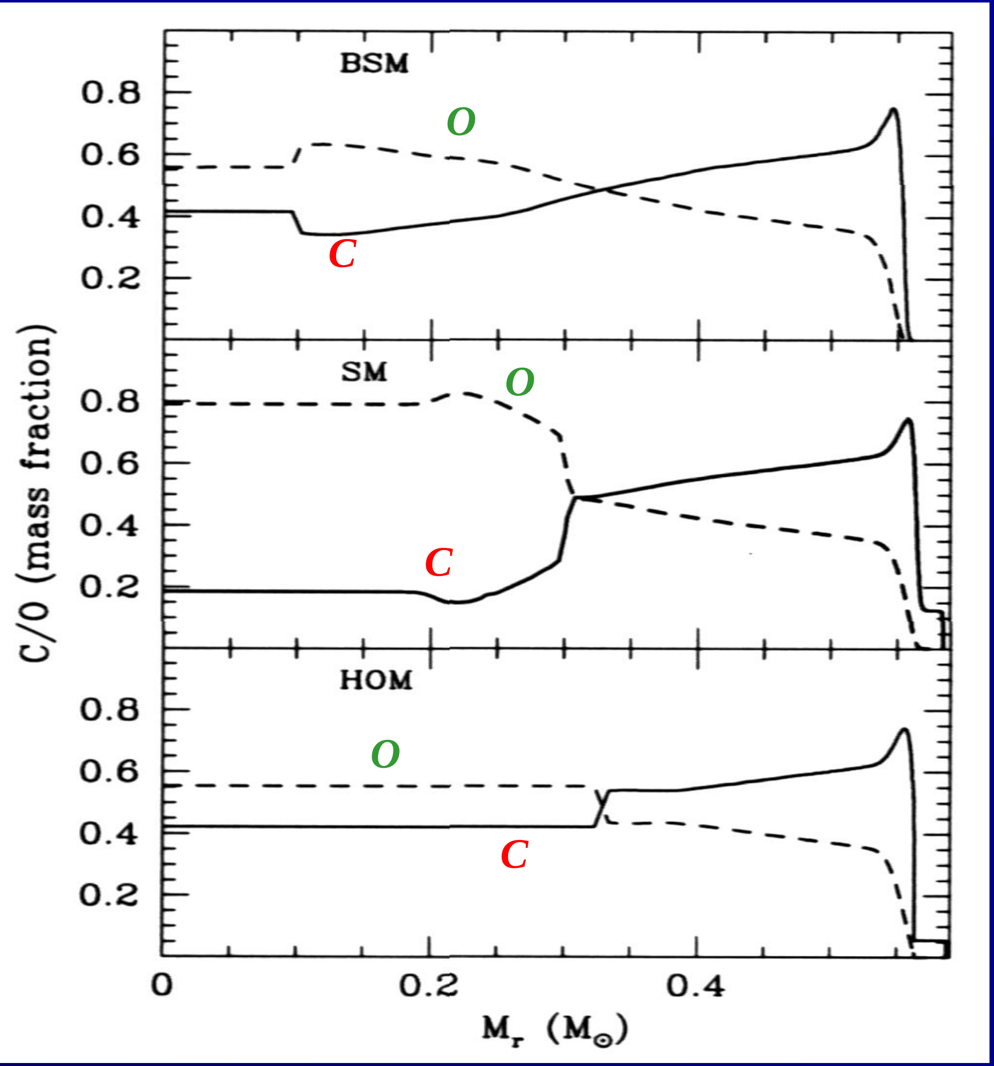
Parameter R2=Nagb/Nhb

Observations R2=0.14±0.02

Semiconv +BPs R2~0.08

Semiconv no BPs R2~0.12





Chemical stratification at the onset of AGB thermal pulse phase

from Straniero et al. (2003)

CONSEQUENCES FOR WD CO ABUNDANCES

Algorithm of BP suppression affects central CO abundances

	τ_{He}	X_C	X_O
BSM	88	0.42	0.56
SM	145	0.19	0.79
PSM	134	0.40	0.58
HOM	153	0.42	0.56
LOM	139	0.38	0.60

From Straniero et al. (2003)

$3M_o$ solar compostion

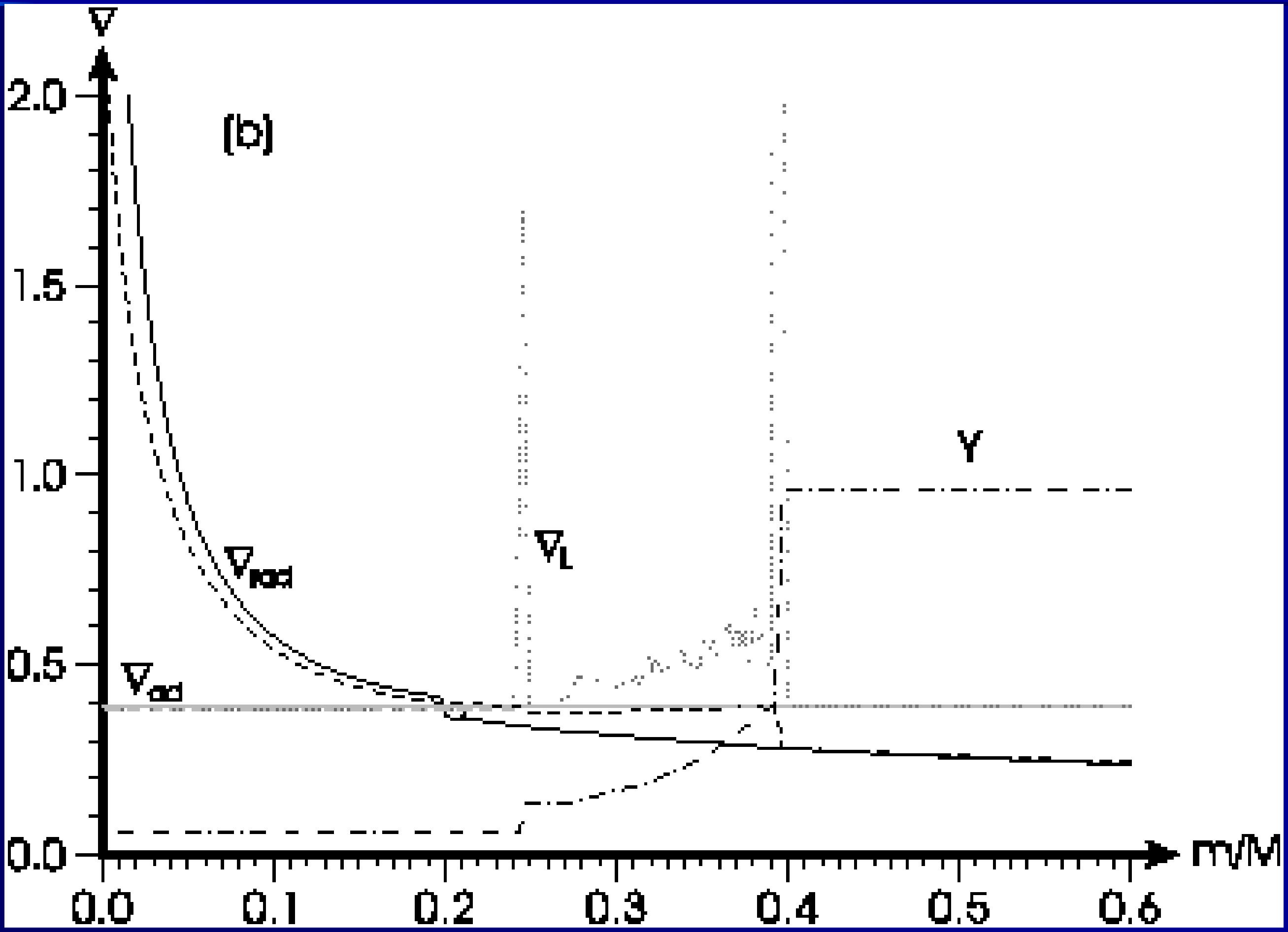
BSM =no oversh., no semiconv.

SM and PSM = semiconv. + 2 different methods to suppress BPs

HOM = overshooting 1Hp (BP suppressed as in SM)

LOM= overshooting 0.2Hp (BP suppressed as in SM)

Diffusive treatment of convection + overshooting (Herwig et al) and semiconvection (Langer)



From Kippenhahn, Weigert & Wess (2012)

Superadiabatic convection

MLT (3 + 1 free parameters) - FST (2 + 1 free parameters)

Non-local   Time-independent

›››› **Affects depth of convective envelopes, T_{eff} , P/rho stratification in the convective envelopes, surface abundances/nucleosynthesis, asteroseismic properties.**

i) Calibration of the MLT parameters (and the FST free parameter) depends on input physics, MLT formalism and boundary conditions chosen.

ii) Uncertainties associated to the calibrators.

iii) Different solar calibrations may provide different results in earlier or later evolutionary phases.

“Semiconvection” + “Overshooting”

Free parameters (or assumptions) related to the extension and efficiency of mixing.

Instantaneous or time-dependent mixing

››››››› **Affect evolutionary times (star counts), luminosities, T_{eff} , loops in the Colour-Magnitude-Diagrams, predicted populations of variable stars in stellar populations, chemical profiles, asteroseismic properties.**

Empirical calibrations are needed.

The calibrated ‘extended’ size of the mixed regions might be due not just to convective overshooting but also to other neglected physical processes and/or inadequacies of the input physics.

How do we move forward ?

Need to keep the number of equations and free parameters to a minimum (ideally no free parameters)

i) Introduce additional ingredients in the MLT (or FST) formalisms (this usually increases the number of free parameters) ? (e.g. Gough 1977, Kuhfuss 1985, Balmforth 1992)

ii) Calibrate the MLT parameter(s) (and overshooting) on 2D or 3D hydro simulations ? (e.g. Ludwig et al. 1999, Freytag & Salaris 1999)

iii) Employ Reynolds stress models ?

(e.g. Xiong 1978, Canuto 1992, 1993, Yang & Li 2007)

… again free parameters/assumptions (Yang & Li model has 6 free parameters)

iv) Other options ?

Diffusive approach

$$\frac{\partial Y}{\partial t} = \left(\frac{\partial Y}{\partial t}\right)_{\rm nuc} + \frac{1}{4\pi r^2 \rho}\frac{\partial}{\partial r} 4\pi r^2 \rho \kappa_Y^* \frac{\partial Y}{\partial r}$$

Fully ionized gas

$$\nabla_{\rm ad} + \frac{\beta}{4-3\beta}\nabla_\mu \equiv \nabla_{\rm L}$$

$$\kappa_Y^* = \alpha_S \frac{\kappa}{6}\frac{\nabla - \nabla_{\rm ad}}{\nabla_L - \nabla}$$

Langer et al. (1983)

$$\kappa_Y^* \lesssim \kappa\left[\left(\frac{4-3\beta}{\beta}\right)\frac{\nabla - \nabla_{\rm ad}}{\nabla_\mu}\right]^2$$

Stevenson (1979)

$$\kappa_Y^* = (\kappa_Y \kappa)^{1/2}\left(\frac{4-3\beta}{\beta}\right)\frac{\nabla_{\rm rad} - \nabla_{\rm ad}}{\nabla_\mu}$$

Spruit (1992)