Bose polarons at finite temperature and strong coupling

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The impurity problem

- Dilute impurities in quantum many-body systems

- Probes of equilibrium and non-equilibrium properties: quasi-particle energies, lifetimes, effective masses, coherence, formation times, orthogonality catastrophe, ...
  - MIT [PRL '09], ENS [PRL '09], Innsbruck [Nature 2012, Science 2017], Cambridge [Nature 2012], LENS [PRL 2018], Aarhus [PRL 2016], JILA [PRL 2016], ...

- Ideal tools to manipulate and study a many-body system (transport, topology, ...)

- Well-posed problem (QMC, variational, diagrammatic, RG, functional determinants, ...)

Outline

✦ Impurities in a weakly-interacting Bose gas ("Bose polarons")

✦ Part I:
  a new quasi-particle appears at non-zero temperature

✦ Part II:
  an Anderson orthogonality catastrophe arises in ideal BECs

Nils Guenther  Maciej Lewenstein  Georg M. Bruun  Richard Schmidt
Impurities in a Bose gas

JILA:  Hu, …, Cornell and Jin [PRL 2016]

Aarhus:  Jørgensen, …, Bruun and Arlt [PRL 2016]

T=0 theory: Rath, Schmidt, Das Sarma, Bruun, Levinsen, Parish, Demler, Peña-Ardila, Giorgini, Pohl, Camacho-Guardian, …

weak RF pulse + quick decoherence: a few $|2\rangle$ impurities in a bath of $|1\rangle$ atoms
Polarons: Fermi vs. Bose

- **Differences**
  - **non-interacting Fermi sea**
    - smooth crossover from degenerate to classical
  - **weakly-interacting Bose gas \((k_n a_B \ll 1)\)**
    - BEC phase transition at \(T_c\)

- **Temperature**
  - smooth crossover from degenerate to classical

- **Impurity ground state**
  - polaron/molecule transition

- **Three-body physics**
  - negligible

- **Stability**
  - (meta-)stable mixture

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[Innsbruck 2012]

[Aarhus 2016]
Part I

Finite-T analysis
Definition of the problem

• Weakly-interacting BEC, treated with Bogoliubov theory
  » condensate density: \( n_0 = n[1 - (T/T_c)^{3/2}] \)
  » critical temperature: \( T_c = \frac{2\pi}{m_B} \left( \frac{n}{\zeta(3/2)} \right)^{2/3} \approx 0.436E_n \)
  » boson-boson vacuum scattering: \( \mathcal{T}_B = 4\pi a_B/m_B \)
  » BEC chemical potential: \( \mu_B = \mathcal{T}_B n_0 \)
  » Bogoliubov excitations: \( E_k = \sqrt{\epsilon_k^B (\epsilon_k^B + 2\mu_B)} \)
  » free bosons: \( \epsilon_k^B = k^2/2m_B \)

• Impurity-bath coupling: s-wave contact interaction with scattering length \( a \)

• Finite temperature Green’s functions:
  • Polaron energy: \( \omega_p = \epsilon_p + \text{Re}[\Sigma(p, \omega_p)] \)
  • Polaron residue: \( Z_p = \frac{1}{1 - \partial_\omega \text{Re}[\Sigma(p, \omega)]|_{\omega_p}} \)

units: \( k_n = (6\pi^2 n)^{1/3} \)
\( E_n = k_n^2/2m_B \)
Diagrammatic scheme

Impurity Green’s function: \( \mathcal{G}(\mathbf{p}, \imath \omega_j) = \frac{1}{\mathcal{G}_0(\mathbf{p}, \imath \omega_j)^{-1} - \Sigma(\mathbf{p}, \imath \omega_j)} \)

\[ \begin{align*}
\mathcal{T} & = \quad + \quad \mathcal{T} \\
\text{Ladder } T\text{-matrix: } \mathcal{T}(\mathbf{p}, \imath \omega_j)^{-1} & = \mathcal{T}_v^{-1} - \Pi(\mathbf{p}, \imath \omega_j)
\end{align*} \]

\[ \begin{align*}
\Sigma & = \quad + \quad \mathcal{T} \\
\text{Extended self-energy: } \Sigma & = \Sigma_0 + \tilde{T} = \mathcal{T}_v + \mathcal{T}_L
\end{align*} \]

at \( T > 0 \), important diagram missing in ladder approx:

\( T = 0 \) ladder: Rath and Schmidt, PRA 2013
Perturbation theory at \( T > 0 \): Levinsen, Parish, Christensen, Arlt, and Bruun, PRA 2017
Extended \( T > 0 \) diagrammatic scheme: Guenther, PM, Lewenstein and Bruun, PRL 2018
Varying coupling strength

\( T=0 \)

\( T=0.5T_c \)

\( T=T_c \)

\[ \omega_{\uparrow} (T=0.5T_c) \]

\[ \omega_{\uparrow}=\omega_{\downarrow} (T=0) \]

\[ \omega_{\downarrow} (T=0.5T_c) \]

\[ \omega_{\downarrow} (T=T_c) \]

Aarhus: \( k_n a_B = 0.01 \)

black diamonds: mean-field

blue dots: \( T=0 \) ladder theory

[Rath and Schmidt, PRA 2013]

Guenther, PM, Lewenstein, and Bruun

Increasing temperature

Weak attraction

\( k_n a = -1 \)

Spectral function:

Guenther, PM, Lewenstein, and Bruun

Increasing temperature

Strong attraction (unitarity)

Energy:

Residue:
Understanding fragmentation

Polaron energy: \( \omega_0 = \text{Re}[\Sigma(p = 0, \omega_0)] \)

\[ \text{Re}(\Sigma) \text{ at weak coupling } \& \text{ low-T:} \]

\[ k_n a = -0.1 \]

\[ \Sigma_1(\omega) \approx \int \frac{d^3 k}{(2\pi)^3} \frac{f_k}{T_v^{-1} - \Pi(k, \omega + E_k) - \frac{n_0}{\omega + E_k - \epsilon_k}} \]

\[ \approx \frac{\omega + n_0 T_B}{\omega - n_0 (T_v - T_B)} n_{\text{ex}} T_v \]

\(|a| \gtrsim a_B : \text{equal splitting}\)

\[ \omega_{\uparrow, \downarrow} \approx \omega_0 [1 \pm (Z_0 n_{\text{ex}}/n_0)^{1/2}] \]

\[ Z_{\uparrow, \downarrow} \approx Z_L/2 \]

\(|a| \lesssim a_B : \text{single polaron}\)

\[ \omega_{\uparrow} \approx n T_v \]

in accord with perturbation theory [Levinsen et al., PRA 2017]
General features

• Strong temperature dependence, due to the Bogolubov spectrum crossing over from linear to quadratic (at $T= T_c$, and at $\epsilon_k^B \approx 2\mu_B$)

• Similar scenario whenever the bath undergoes a phase transition breaking a continuous symmetry

• Examples: normal and high-$T_c$ superconductors, $^3\text{He}-^4\text{He}$ mixtures, ultracold fermionic superfluids, magnetic systems, …

• The new quasiparticle emerges due to the coupling between the impurity and a large number of low-energy soft excitations (like Landau damping in ordinary plasmas, and plasminos in Yukawa and QED theories)

Baym, Blaizot, and Svetitksy, PRD 1992
Validity of the 1PH-approx for an ideal BEC?

equal masses at unitarity

Energy

Residue

Rath and Schmidt, PRA 2013

Yoshida, Endo, Levinsen and Parish, PRX 2018
Part II

Infinitely-massive impurities
Ideal BEC

- Imp+boson inside a sphere of radius $R$; non-interacting: $\psi(r) = \frac{1}{\sqrt{2\pi R}} \frac{\sin(k_0 r)}{r}$

- Adding a short-ranged interaction: $\psi(r) = \frac{1}{\sqrt{2\pi R}} \frac{\sin(k r + \delta)}{r}$

- Energy shift: $\Delta E = \frac{\hbar^2}{m} k \Delta k = \frac{2\pi \hbar^2}{m_r} \left( -\frac{\delta}{k} \right) |\psi(0)|^2$

- Phase shift: $\cot \delta = -\frac{1}{k a} + \frac{r_e k}{2} + O(k^3)$

- Weak interaction: $\delta = -ka$ (MF shift)

- Unitarity: $\delta = \pi/2$
  taking $k = 1/\xi$ one has $\Delta E = -\frac{1}{3} (k_n \xi) E_n$
  which diverges for an ideal gas!
Ideal BEC

- The BEC is a product state: $|\Psi\rangle = \prod_{N} |\psi\rangle$

- If $|\psi\rangle \neq |\psi_0\rangle$, then $z = |\langle\psi_0|\psi\rangle|^2 < 1$

- And in the thermodynamic limit the residue $Z = z^N \to 0$

- For every bath-impurity interaction!

- Ideal BEC + $\infty$-mass impurity $\Rightarrow$ Anderson orthogonality catastrophe
Weakly-interacting BEC

- GPE equation for the radial wavefunction \( \psi(r) = \frac{\chi(r)}{r} \)

- BEC-impurity potential: attractive square well

- B.C. : \( \chi(0) = \chi''(0) = 0 \) and \( \chi(r \to \infty) = r\sqrt{n_0} \)

- Energy shift:

\[
\frac{\Delta E}{E_n} = \frac{k_n a_B}{1/k_n a}
\]

\( n \) vs \( \frac{1}{k_n a} \) for different values of \( k_n a_B \): 
- \( k_n a_B = 0.01 \)
- \( k_n a_B = 0.1 \)
- \( n T_v \)
Weakly-interacting BEC

- Number of particles in the dressing cloud: \[ \Delta N = \int \mathrm{d} \mathbf{r} \left[ n(\mathbf{r}) - n_0 \right]. \]

- For \(|a| \lesssim a_B\) one finds \[ \Delta N = -\frac{a}{2a_B} \]

Massignan, Pethick and Smith, PRA 2005
Weakly-interacting BEC

- $\psi(\mathbf{r}) = \psi_0 + \delta\psi(\mathbf{r})$

- Overlap of normalized GPE solutions: 
  $$z \equiv \left| \frac{\langle \psi_0 | \psi \rangle}{\sqrt{N_0 N}} \right|^2 = 1 - \frac{c}{N_0} + \mathcal{O}(N_0^{-2})$$

  where $c \equiv \int d\mathbf{r} [\delta\psi(\mathbf{r})]^2 > 0$

- Many-body overlap: 
  $$Z = \lim_{N_0 \to \infty} z^{N_0} = \lim_{N_0 \to \infty} \left( 1 - \frac{c}{N_0} + \mathcal{O}(N_0^{-2}) \right)^{N_0} = e^{-c}$$

- Exponentially small residue $\Rightarrow$ Anderson orthogonality catastrophe!

$$Z = e^{-\alpha N^{1/3}} = e^{-\beta n_0 a^2 R}$$
Weakly-interacting BEC

- Linearized GPE: \( \psi_{\text{lin}}(r) = \sqrt{n_0} \left[ 1 + C \frac{\exp(-\sqrt{2}r/\xi)}{r} \right] \)

- \( Z_{\text{lin}} = e^{-c_{\text{lin}}} \approx 1 - \sqrt{2}\pi n_0 a^2 \xi \) very close to \( Z_{\text{pert}} = 1 - 4\sqrt{2}n_0 a^2 \xi + \mathcal{O}(n_0 a^3) \)

Christensen, Levinsen & Bruun, PRL 2015
Conclusions

• Bose polarons greatly differ from Fermi ones
• Crucial role played by the large low-energy density of states of the BEC
• Non-perturbative treatment is fundamental
• The $T=0$ attractive polaron fragments into two quasiparticles at $T>0$
• The ground state quasiparticle remains well-defined across $T_c$
• The excitation above it instead disappears at $T_c$

• An *Anderson orthogonality catastrophe* arises when $a_B \to 0$

Thank you!