Bose polarons
at finite temperature
and strong coupling

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Outline

✦ Quantum mixtures

✦ Impurities in an ideal Fermi sea ("Fermi polarons")

✦ Impurities in a weakly-interacting Bose gas ("Bose polarons")

✦ Zero-temperature physics

✦ New features appearing at non-zero temperature
Collaborators

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Quantum Mixtures in CondMat

- $^3$He-$^4$He
- ultracold gaseous mixtures:
  - FF (BEC-BCS crossover)
  - B+FF superfluids (coherent/damped dynamics)
  - BB mixtures (ultradilute quantum liquid droplets)
  - spinor gases, SU(N) invariant systems
- quantum magnets, quantum Hall systems, spin-liquids
- quark-gluon plasma
- neutron stars

Very different microscopically, but
∃ common **emergent and universal** many-body descriptions.
Universality in Quantum Mixtures

20 orders of magnitude difference in temperature

but similar transport properties!

e.g., shear viscosity/entropy density:

[Adams et al., NJP 2012]
Many-body systems

A GUIDE TO FEYNMAN DIAGRAMS

Angels on pinhead

Nucleons in nucleus
Electrons in atom
Atoms in molecule
Atoms in solid

Molecules in liquid
Electrons in metal

Fig. 0.1 Some Many-body Systems

(from Richard Mattuck’s book)
Quasi-Particles

No chance of studying real particles

Landau:
of importance are the collective excitations,
which generally behave as quasi-particles!

a QP is a “free” particle with:
@ q. numbers (charge, spin, ...)
@ renormalized mass
@ chemical potential
@ shielded interactions
@ lifetime
Ultracold atoms

- chemical composition
- interaction strength
- temperature

- periodic potentials
- physical dimension
- atom-light coupling
- exotic interactions
  (x-wave, spin-orbit)
- dynamics
- disorder
- periodic driving
  (shaken optical lattices)
Imbalanced Fermi gases

Two-component Fermi gas with $N_{\uparrow} \gg N_{\downarrow}$: a strongly-interacting system, or an ensemble of weakly-interacting quasi-particles (a Fermi liquid)

$$E = \frac{3}{5} \epsilon_F N_{\uparrow} \left[ 1 + \frac{m}{m^*} \left( \frac{N_{\downarrow}}{N_{\uparrow}} \right)^{5/3} \right] + N_{\downarrow} E_p + \ldots,$$

- kinetic energy of the Fermi sea
- kinetic energy of the polarons ($m^*$ is their effective mass)
- chemical potential (energy) of one polaron
Spectrum of Fermi polarons

low power RF:

high power RF:

high power is needed to couple to the MH continuum, due to a small FC overlap

thory: Chevy, Recati, Combescot, Zwerger, Enss, Schmidt, Bruun, Pethick, Zhai, Levinsen, Parish, Castin, …
Repulsive polarons

- most general case
  (equal masses, broad resonance)

- \( \exists \) meta-stable quasiparticle at \( E>0 \)

- long-lived, even close to unitarity

- measures of \( E, m_{\text{eff}} \) and \( Z \)
  in very good agreement with simple theory

- polaron-polaron interactions negligible

[Scazza, Valtolina, PM, Recati et al., PRL 2017]
Impurities in a Bose gas

weak RF pulse + quick decoherence: a few $|2\rangle$ impurities in a bath of $|1\rangle$ atoms

JILA: Hu, …, Cornelll and Jin, PRL (2016)

Aarhus: Jørgensen, …, Bruun and Arlt, PRL (2016)

T=0 theory: Rath, Schmidt, Das Sarma, Bruun, Levinsen, Parish, Giorgini, …
# Fermi vs. Bose

<table>
<thead>
<tr>
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<th>non-interacting Fermi sea</th>
<th>weakly-interacting Bose gas ((k_{\text{NaB}} \ll 1))</th>
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<tr>
<td><strong>Temperature</strong></td>
<td>smooth crossover from degenerate to classical</td>
<td>BEC phase transition at (T_c)</td>
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<td><strong>Impurity ground state</strong></td>
<td>polaron/molecule transition</td>
<td>smooth crossover</td>
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<td><strong>Three-body physics</strong></td>
<td>negligible</td>
<td>important role</td>
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<td><strong>Stability</strong></td>
<td>rather stable mixture</td>
<td>rapid three-body losses</td>
</tr>
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Definition of the problem

• Bath treated with Bogoliubov theory
  » critical temperature: \( T_c = \frac{2\pi}{m_B} \left( \frac{n}{\zeta(\frac{3}{2})} \right)^{2/3} \approx 0.436E_n \)
  » condensate density: \( n_0 = n[1 - (T/T_c)^{3/2}] \)
  » bath chemical potential: \( \mu_B = T_B n_0 \)
  » bath vacuum scattering matrix: \( T_B = 4\pi a_B / m_B \)
  » dispersion of the excitations: \( E_k = \sqrt{\epsilon_k^B(\epsilon_k^B + 2\mu_B)} \)
  » free bosons: \( \epsilon_k^B = k^2 / 2m_B \)

• Impurity-bath coupling: finite temperature Green’s functions (non-perturbative!)
  • Polaron energy: \( \omega_p = \epsilon_p + \text{Re}[\Sigma(p, \omega_p)] \)
  • Polaron residue: \( Z_p = \frac{1}{1 - \partial_\omega \text{Re}[\Sigma(p, \omega)]|_{\omega_p}} \)

units: \( k_n = (6\pi^2 n)^{1/3} \)
\( E_n = k_n^2 / 2m_B \)
Diagrammatic scheme

Impurity Green’s function: \( \mathcal{G}(\mathbf{p}, i\omega_j) = \frac{1}{\mathcal{G}_0(\mathbf{p}, i\omega_j)^{-1} - \Sigma(\mathbf{p}, i\omega_j)} \)

Ladder T-matrix: \( \mathcal{T}(\mathbf{p}, i\omega_j)^{-1} = \mathcal{T}_v^{-1} - \Pi(\mathbf{p}, i\omega_j) \)

Extended T-matrix: \( \tilde{\mathcal{T}}(\mathbf{p}, i\omega_j)^{-1} = \mathcal{T}_1 v^{\uparrow}(\mathbf{p}, i\omega_j) + \sum_n \mathcal{G}_0(\mathbf{p}, i\omega_j) \)

Extended self-energy: \( \Sigma = \Sigma_0 + \tilde{\mathcal{T}} \bigg| \{z\} \Sigma_1 + \tilde{\mathcal{T}} \bigg| \{z\} \Sigma_L + \tilde{\mathcal{T}} \)

T=0 ladder: Rath and Schmidt, PRA 2013
Perturbation theory at T>0: Levinsen, Parish, Christensen, Arlt, and Bruun, arXiv:1708.09172
Extended T>0 diagrammatic scheme: Guenther, PM, Lewenstein and Bruun, arXiv:1708.08861
Varying coupling strength

$T=0$
$T=0.5T_c$
$T=T_c$

Aarhus: $k_n a_B = 0.01$

dots: $T=0$ ladder theory by Rath and Schmidt, PRA 2013

Guenther, PM, Lewenstein, and Bruun, arXiv:1708.08861
Varying temperature

Weak attraction
\( k_n a = -1 \)

Energy:

Residue:

Guenther, PM, Lewenstein, and Bruun, arXiv:1708.08861
Varying temperature

Strong attraction (unitarity)

Energy:

Residue:

Guenther, PM, Lewenstein, and Bruun, arXiv:1708.08861
Understanding fragmentation

\[ \omega_0 = \text{Re}[\Sigma(p = 0, \omega_0)] \]

Weak coupling,

low temperature behavior of \( \text{Re}(\Sigma) \):

\( (k_n a = -0.1) \)

\[ \Sigma_1(\omega) \approx \int \frac{d^3k}{(2\pi)^3} \frac{f_k}{T_v^{-1} - \Pi(k, \omega + E_k) - \frac{n_0}{\omega + E_k - \epsilon_k}} \]

\[ \approx \frac{\omega + n_0 T_B}{\omega - n_0(T_v - T_B)} n_{\text{ex}} T_v \]

non-condensed fraction

\( |a| \gtrsim a_B : \text{equal splitting} \)

\[ \omega_{\uparrow, \downarrow} \approx \omega_0[1 \pm (Z_0 n_{\text{ex}} / n_0)^{1/2}] \]

\[ Z_{\uparrow, \downarrow} \approx Z_L / 2 \]

\( |a| \lesssim a_B : \text{single polaron} \)

\[ \omega_{\uparrow} \approx n T_v \]

in accord with perturbation theory [Levinsen et al., arXiv:1708.09172]
Repulsive polarons

Weak repulsion
\((k_n a = 1)\)

Spectral function:

\[
A(k=0, \omega)E_n
\]

Guenther, PM, Lewenstein, and Bruun, arXiv:1708.08861
Conclusions

• Bose polarons greatly differ from Fermi ones
• No polaron/molecule transition
• Fundamental role played by the BEC, and the associated large low-energy density of states
• Non-perturbative treatment is crucial
• The $T=0$ attractive polaron fragments into two quasiparticles at $T>0$
• The upper of the two negative energy excitations disappears at $T_c$
• The ground state quasiparticle remains well-defined across $T_c$

Thank you!