Glass to superfluid transition in dirty bosons on a lattice

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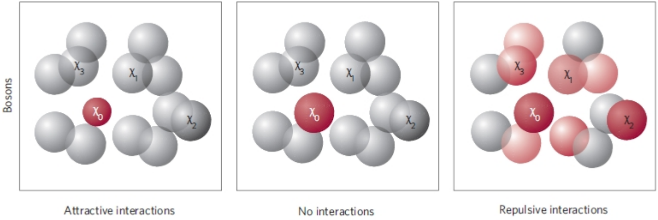
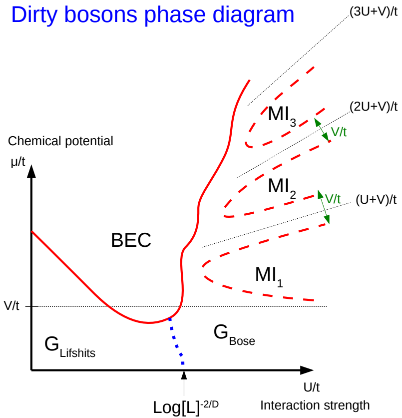
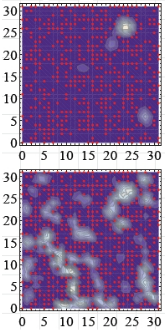
General properties of weakly-interacting disordered bosonic systems



Phase diagram

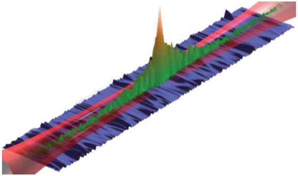




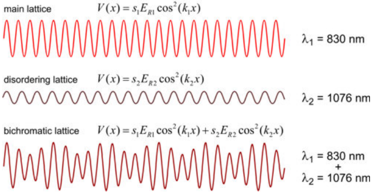
		Superfluid	Compressible	Gapless	Fragmented
BEC		Y	Y	Y	N
Glass	Lifshits	N	Y	Y	Y
	Bose	N	Y	Y	N
Mott insulator		N	N	N	N

Disorder on a lattice

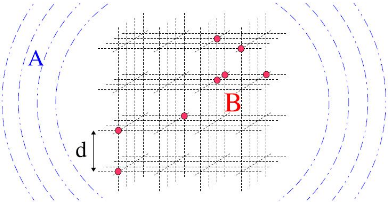
Speckle potential



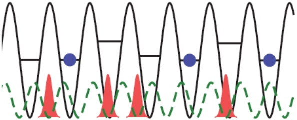
Pseudo-random bichromatic lattice



Random impurities: Bernoulli potential



[P. Massignan and Y. Castin, PRA **74**, 013616 (2006)]

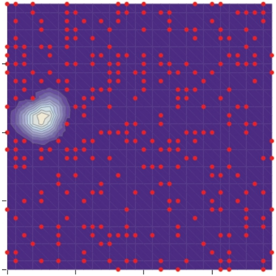


[B. Gadway *et al.*, PRL **107**, 145306 (2011), *Glassy behavior in a binary atomic mixture*.]

Advantages

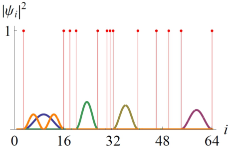
The potential in each lattice site is an *independent random variable* with Bernoulli distribution

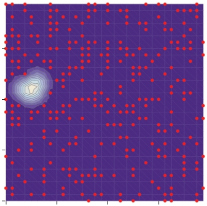
$$\hat{V} = \begin{cases} 0 & \text{with probability } p \\ V > 0 & \text{with probability } 1-p. \end{cases}$$



- ✓ simple form: allows for analytical estimates
- ✓ optimal convergence properties: asymptotic behaviour become visible for fairly small systems
- ✓ requires sampling few potential realizations

Non-interacting Bose gas





1D:

D-dimensional:

- length of the largest island of zero-potential

$$L_{\max}^{(1D)} \propto \log L \qquad L_{\max}^{(D)} \propto (\log L)^{1/D}$$

- energy of the ground state

$$E_0^{(1D)} \propto 1/(\log L)^2 \qquad E_0^{(D)} \propto 1/(\log L)^{2/D}$$

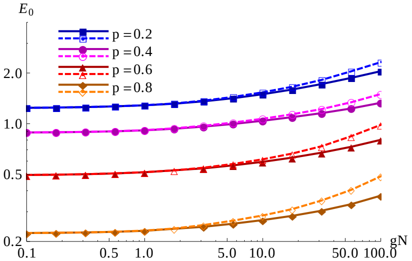
[M. Bishop and J. Wehr, arXiv: math-ph/1109.4109]

Description of weakly repulsive Bose gas

Multi-orbital Hartree-Fock (MOHF): expansion into non-interacting eigenstates

$$E_0 = \sum_\imath n_k E_k + \frac{g}{2} \sum_{k,l} \left[n_k(n_k - 1) O_{kk} + \sum_{l \neq k} 2 n_k n_l O_{kl} \right],$$

E_k - kth eigenstate energy
n_k - occupation numbers, $\sum_k n_k = N$
g - interaction constant
$O_{kl} = \sum_\imath |\psi_\imath^{(k)}|^2 |\psi_\imath^{(l)}|^2$

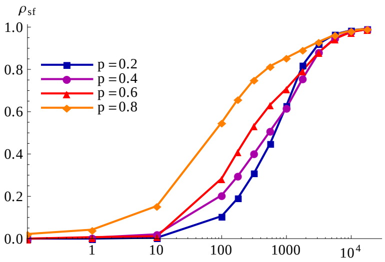


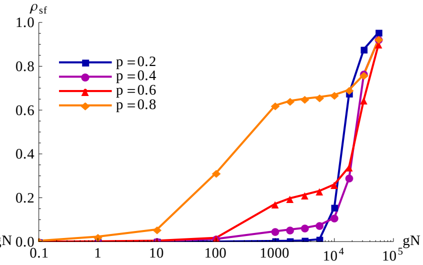
Comparison of obtained energies with GP results

Superfluid fraction

$$\rho_{\text{sf}} = \frac{2mL^2}{\hbar^2}\frac{E(\Phi)-E(0)}{\Phi^2}$$

$E(\Phi)$ - energy of a system with the total phase shift Φ





V=5 (weak disorder) V=50 (strong disorder)

Fractal dimension and fractional occupation

Fractal dimension:
minimum d^* such that:

$$\lim_{L\to\infty} \frac{P}{L^{d^*}} = c,\ c > 0$$

$P = 1/\int \mathrm{dx} |\psi(\mathrm{x})|^4$ - participation number
$\sim$ volume occupied by the state

For Bernoulli potential:

non-interacting localized states

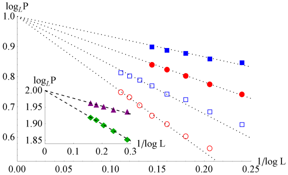
$$d^* = 0$$

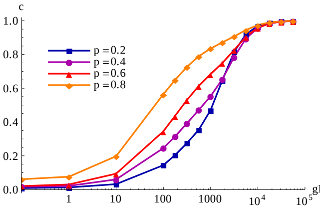
with interaction

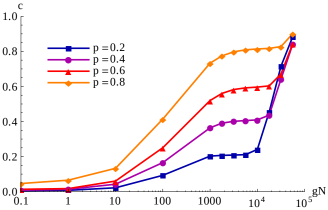
$$d^* = D$$

Fractional occupation:
fraction of space c occupied by the state

$$\log_L P = D - \frac{\log c}{\log L}$$



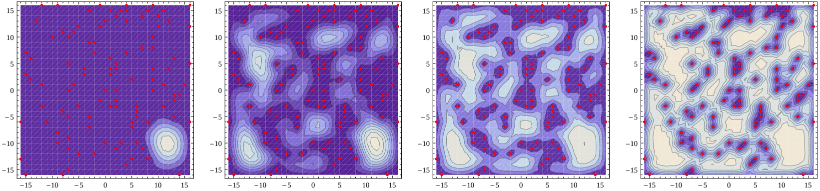




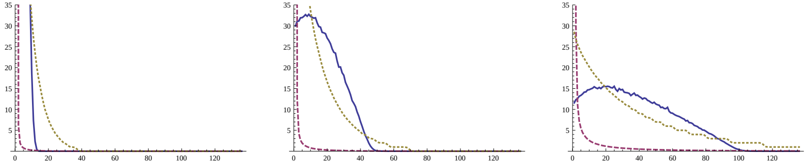
Temperature vs. interaction

Temperature yields effects similar to those of repulsive interaction.

Similarities at the level of particle density:

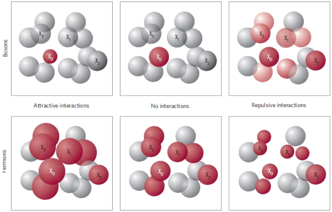


Clear differences at the level of distribution



Conclusions and prospects

- ✓ Study of repulsive interactions between bosons in Bernoulli potential using MOHF and GP
- ✓ Multi-orbital Gross-Pitaevskii for attractive interactions: ongoing project in collaboration with Laurent Sanchez-Palencia
- ✓ Fermionic systems



J Stasinska *et al.*, New. J. of Phys. **14**, 043043 (2012)
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