

# SYNTHETIC GAUGE FIELDS FOR ULTRACOLD ATOMS IN SYNTHETIC DIMENSIONS

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# Phase transitions

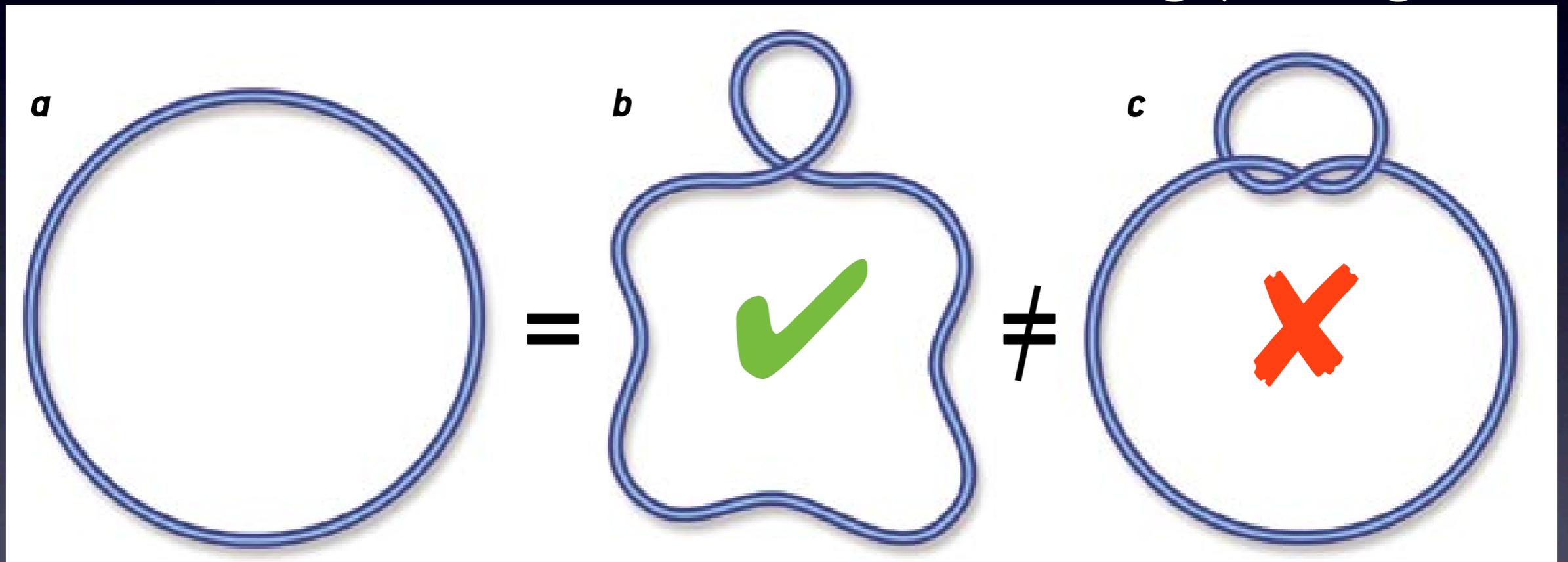
- **Landau**: most phases of matter may be classified by the symmetries they break
  - ▶ translational (solids)
  - ▶ rotational (magnets)
  - ▶ gauge (superfluids)
- **BUT**: some materials possess distinguishable phases without breaking symmetries  
(QH and QSH effect)

Topological phase transitions!

# Topological properties

✓: stretching, bending

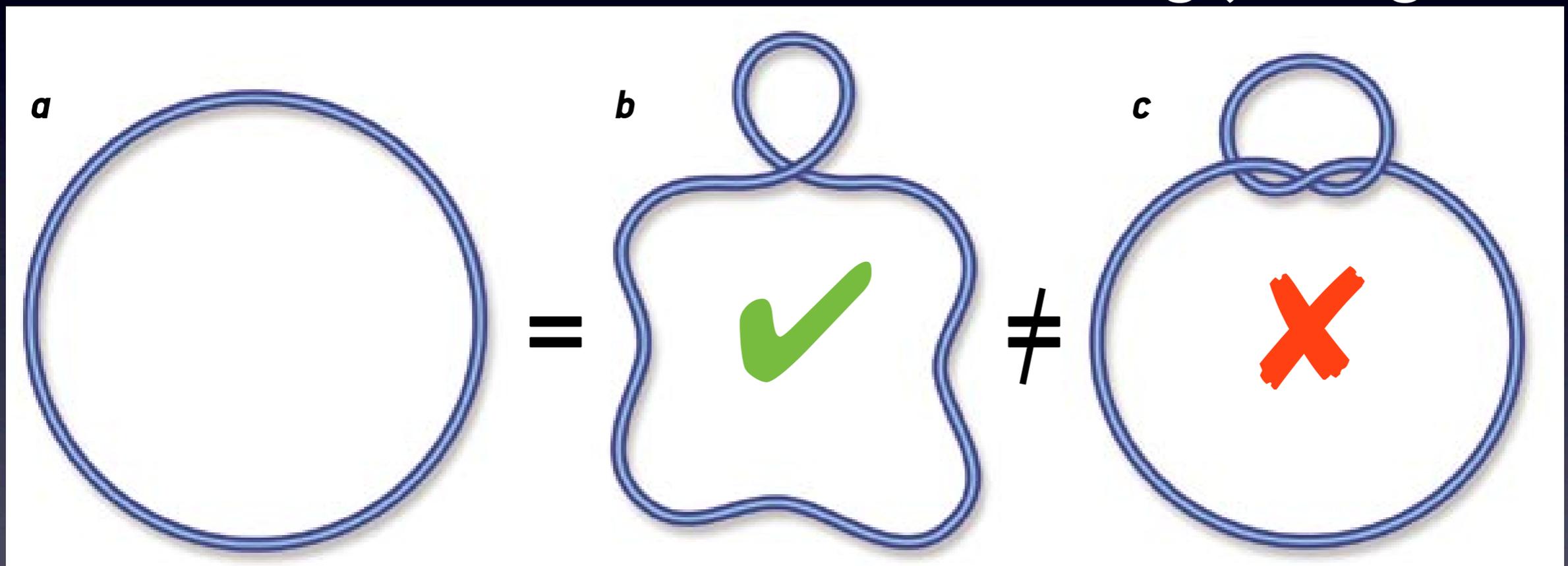
✗: cutting, joining



# Topological properties

✓: stretching, bending

✗: cutting, joining



Concern the whole system (non-local)

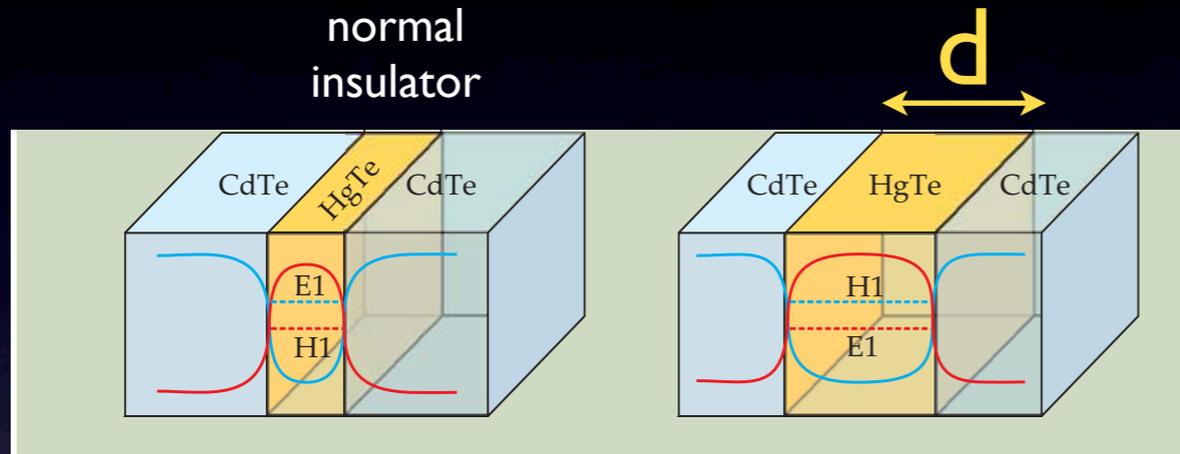
Characterized by integer numbers

Robust

# A topological insulator

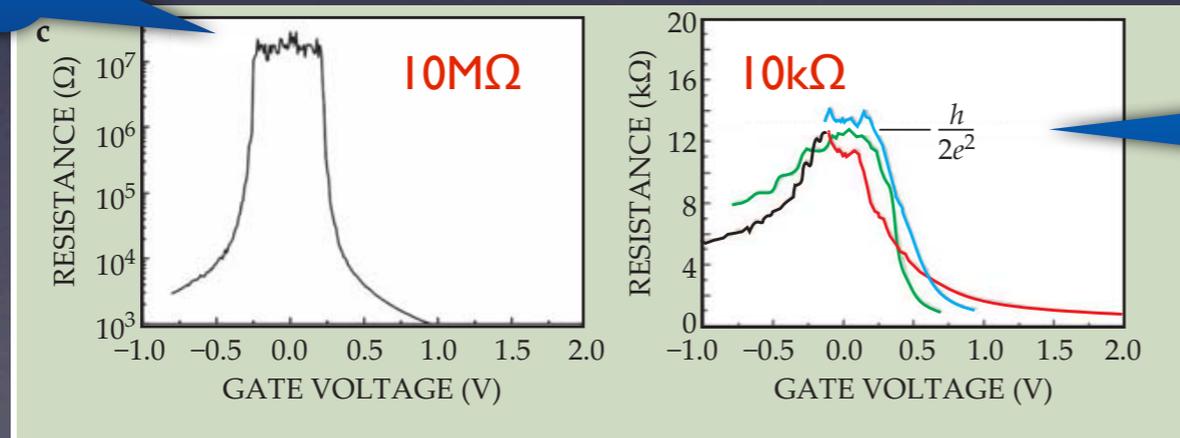
## Hg-Te quantum well

Hg: Mercury  
Te: Telluride



Phase transition at  $d=d_{\text{crit}}$ :  
normal-to-topological insulator

very large  
resistance



independent of  $d$ , when  $d > d_{\text{crit}}$   
2 quanta of conductance

Qi & Zhang, Physics Today 2010

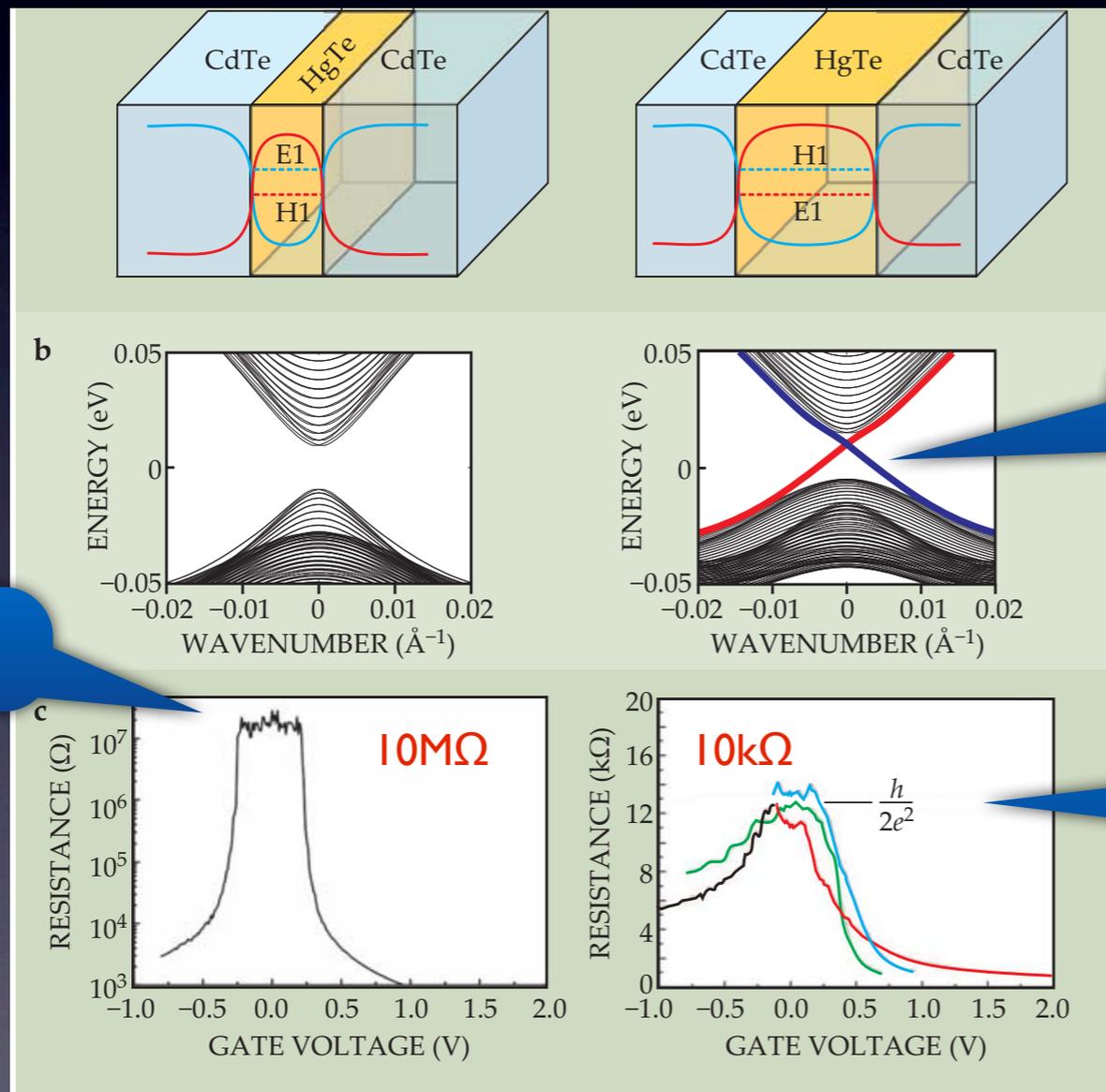
# A topological insulator

## Hg-Te quantum well

Hg: Mercury  
Te: Telluride

normal  
insulator

$d$



$d > d_{\text{crit}}$ : topological insulator

edge states

Hg-Te has strong  
spin-orbit coupling

very large  
resistance

2 quanta of conductance  
(independent of  $d$ , when  $d > d_{\text{crit}}$ )

Qi & Zhang, Physics Today 2010

# interesting..., but where?

- exotic condensed matter systems  
(quantum wells, bismuth antimony alloys,  $\text{Bi}_2\text{Se}_3$  crystals, ...)
- $\nu=5/2$  FQH state (Pfaffian)
- ultracold atoms?

# Outlook of the talk

Synthetic gauge fields

$\uparrow\downarrow$  2D s-wave fermionic SF  
with  $n_{\uparrow} \neq n_{\downarrow}$   
and spin-orbit coupling

Synthetic dimensions

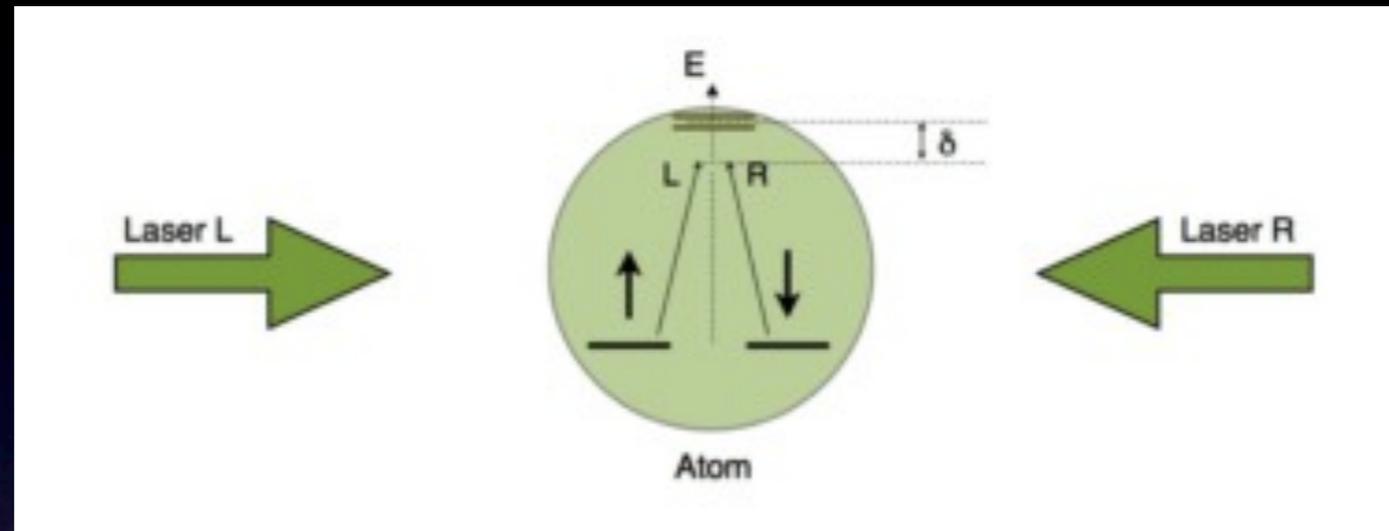
# Synthetic gauge fields for neutral atoms

Theory: Jaksch&Zoller, NJP 2003  
Osterloh et al., PRL 2005  
Gerbier&Dalibard, NJP 2010  
Bermudez et al., PRL 2010 (TRI Top. Ins.)

- adiabatic Raman passage
- adiabatic control of superpositions of degenerate dark states
- spatially varying Raman coupling
- Raman-induced transitions to auxiliary states in optical lattices

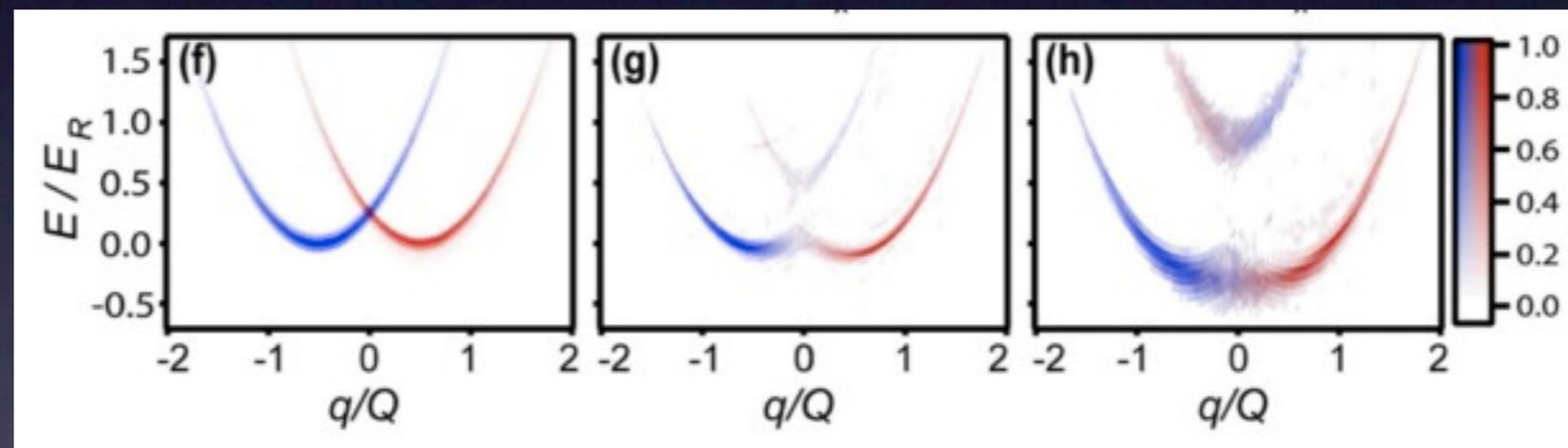
**REVIEW:** *Artificial gauge potentials for neutral atoms*  
J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, RMP 2011

# Synthetic gauge fields for neutral atoms



$$|\uparrow, q=k_x - Q/2\rangle$$

$$|\downarrow, q=k_x + Q/2\rangle$$



spin-orbit gap

increasing intensity of Raman lasers

spin flip  $\leftrightarrow$  momentum kick,  
i.e., spin-orbit coupling

P.Wang et al., PRL 2012 (Shanxi U.)  
L.W. Cheuk et al., PRL 2012 (MIT)

# a field moving fast..

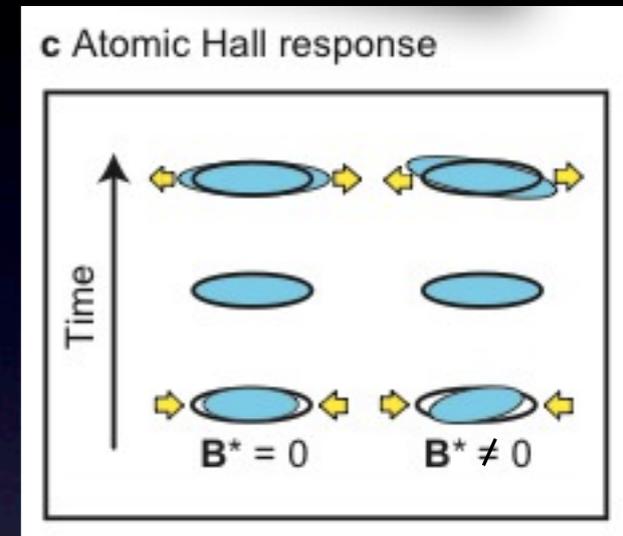
NIST: *Synthetic magnetic fields for ultracold neutral atoms*, Nature (2009)

*A synthetic electric force acting on neutral atoms*, Nature Phys. (2011)

*Spin-orbit-coupled Bose-Einstein condensates*, Nature (2011)

*Observation of a superfluid Hall effect*, PNAS (2012)

*Peierls Substitution in an Engineered Lattice Potential*, PRL (2012)



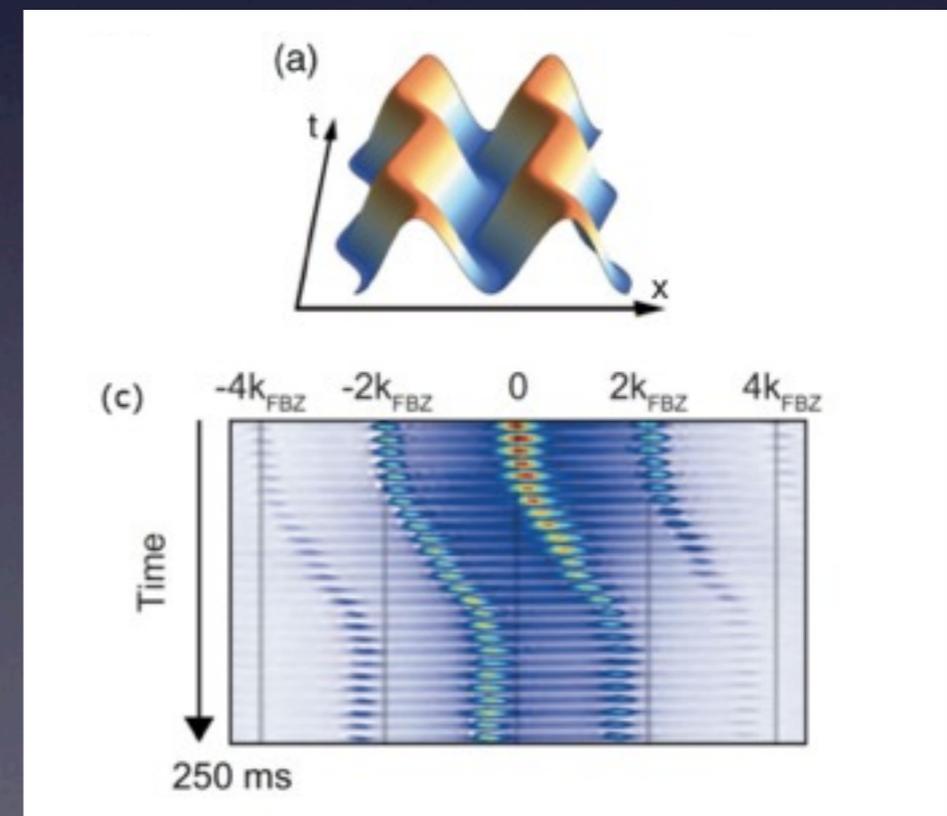
ICFO & Hamburg & Dresden:

*Tunable Gauge Potential for Neutral Spinless Particles in Driven Optical Lattices*, PRL (2012)

(method independent of the internal structure of the atoms!!)

Munich: *Experimental realization of strong effective magnetic fields in an optical lattice*, PRL (2011)

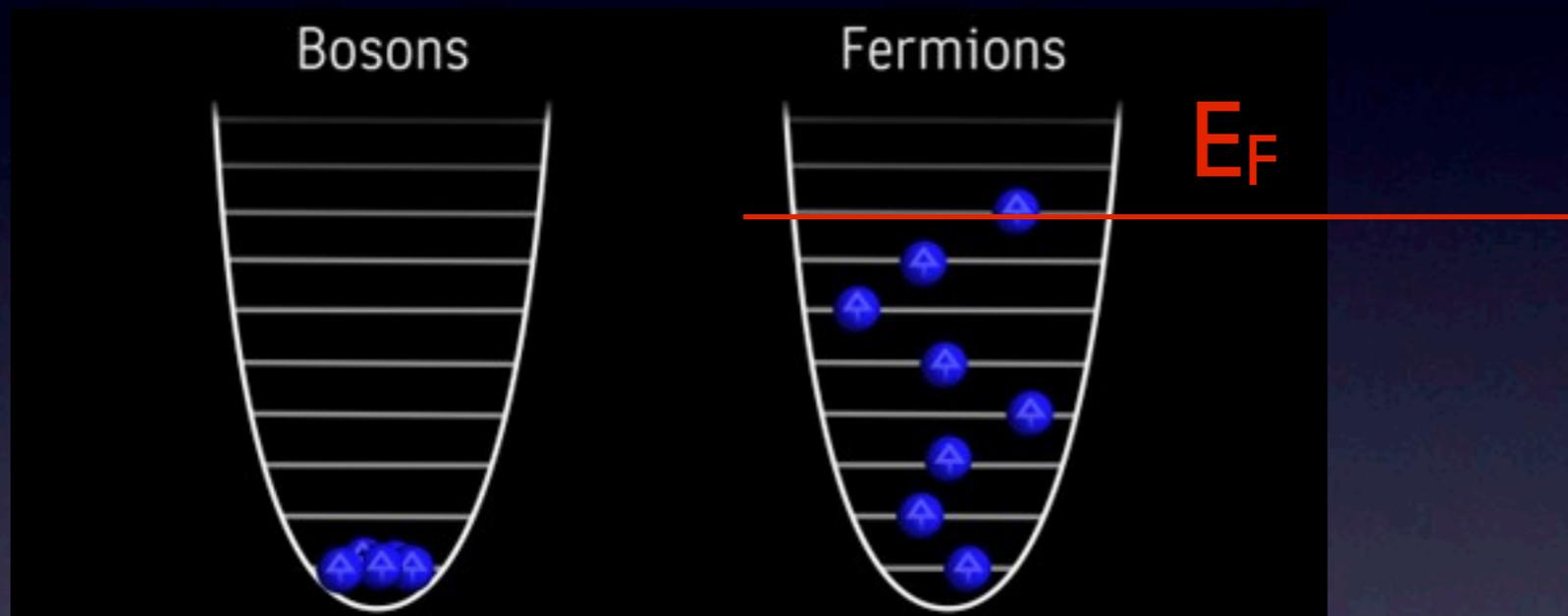
.....



$\uparrow$ - $\downarrow$  fermionic SF  
with  $n_{\uparrow} \neq n_{\downarrow}$   
and spin-orbit coupling

# Fermions vs. Bosons

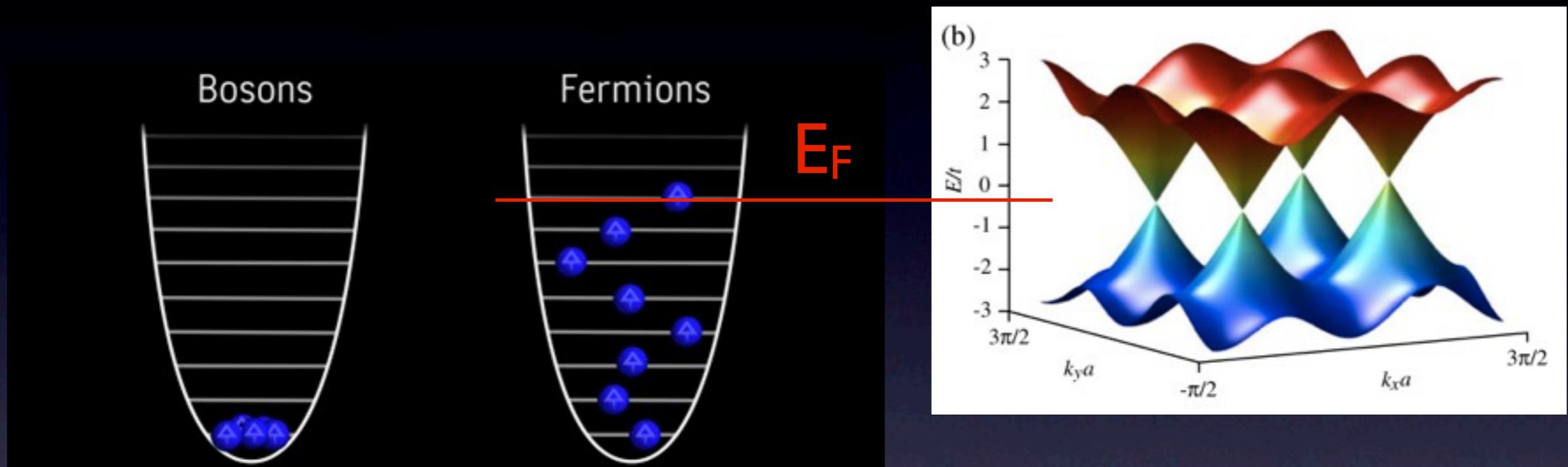
Bosons condense in the lowest available energy state.



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# Fermions vs. Bosons

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On the contrary, fermions have to due to obey the Pauli principle.

By changing the number of particles, we are able to investigate the interesting excitations: the system becomes sensitive to the topological properties of the band structure.

# $\uparrow\downarrow$ fermions in synthetic gauge fields

$$\mathbf{c}_i^\dagger = (c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger)$$

$$\mathcal{H}_0 = -t \sum_i \left[ c_{i+\hat{x}}^\dagger e^{i\sigma_y \alpha} c_i + c_{i+\hat{y}}^\dagger e^{i\sigma_x \beta} c_i + \text{h.c.} \right]$$

complex hoppings = Peierl's phases

External non-Abelian gauge fields yield a **fictitious spin-orbit coupling**

# Add attractive interactions



## BCS superfluid



Sato, Takahashi & Fujimoto, PRL 2009

Sau Jay, Lutchyn, Tewari and Das Sarma, PRL 2010

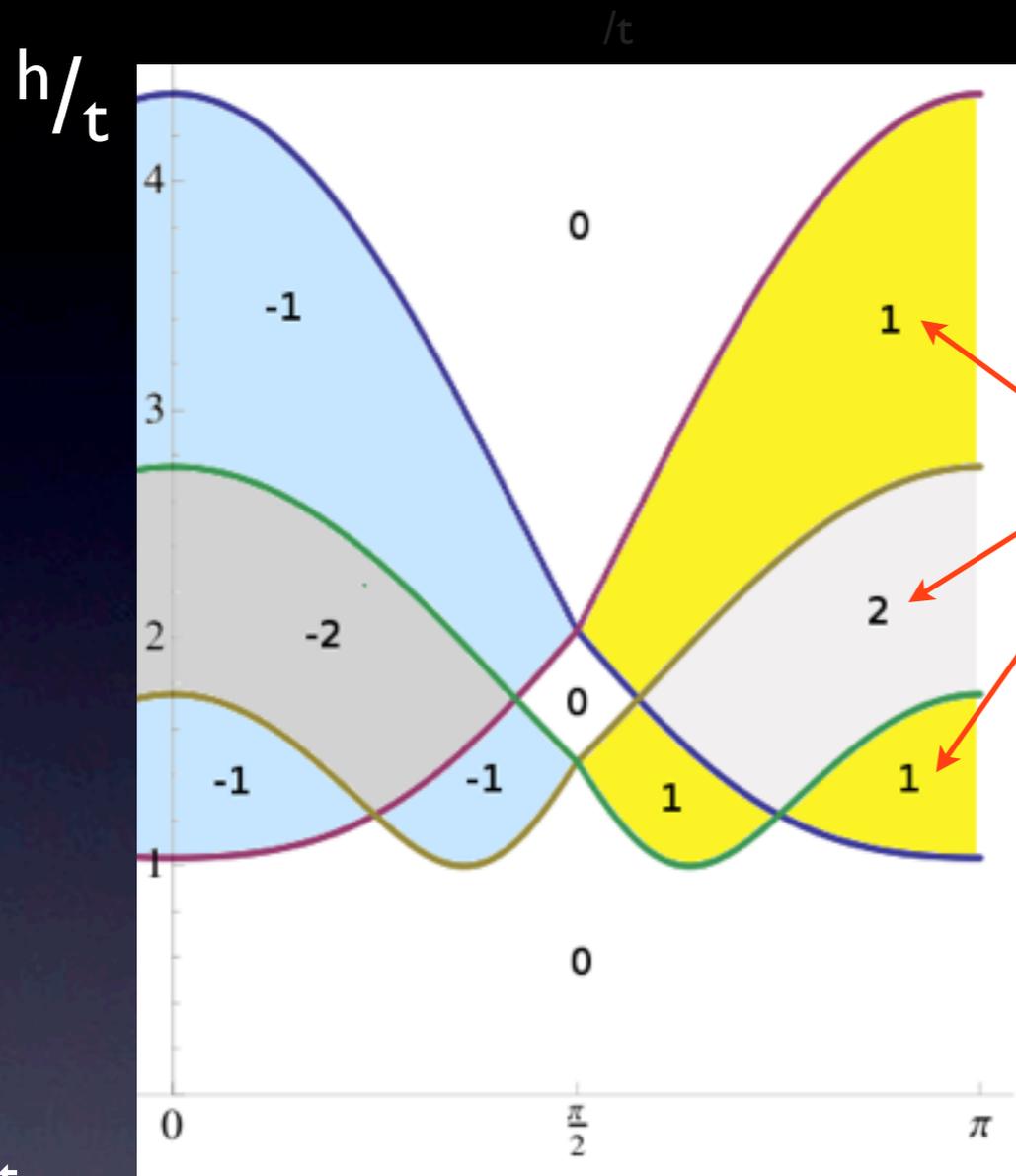
## strong imbalance $\Rightarrow$ topological states

Time-reversal and spin-rotation invariances are destroyed by the Zeeman and SO terms as a consequence our BCS Hamiltonian belongs to the most general symmetry class “D”

(Altland&Zirnbauer, PRB 1997)

its topological phases are indexed in terms of an integer number

# Topological phases



$$h = \mu_{\uparrow} - \mu_{\downarrow}$$

Chern numbers

easy to calculate!

(see J. Bellissard, condmat/9504030)

Gap closing at  $(\mathbf{k}_0, \tilde{h})$ :

$$\mathcal{H}_{\text{eff}}(\mathbf{k}, h) = E(\mathbf{k}, h) + \vec{\sigma} \cdot \vec{f}(\mathbf{k}, h)$$

$$\Delta \text{CN}(\tilde{h}) = \text{sign}\{\det[J_{\vec{f}}(\mathbf{k}_0, \tilde{h})]\}.$$

$\beta$

$$\Delta = t$$

$$\alpha = \pi/4$$

$$\mu = -0.5t[|\cos(\alpha)| + |\cos(\beta)|]$$

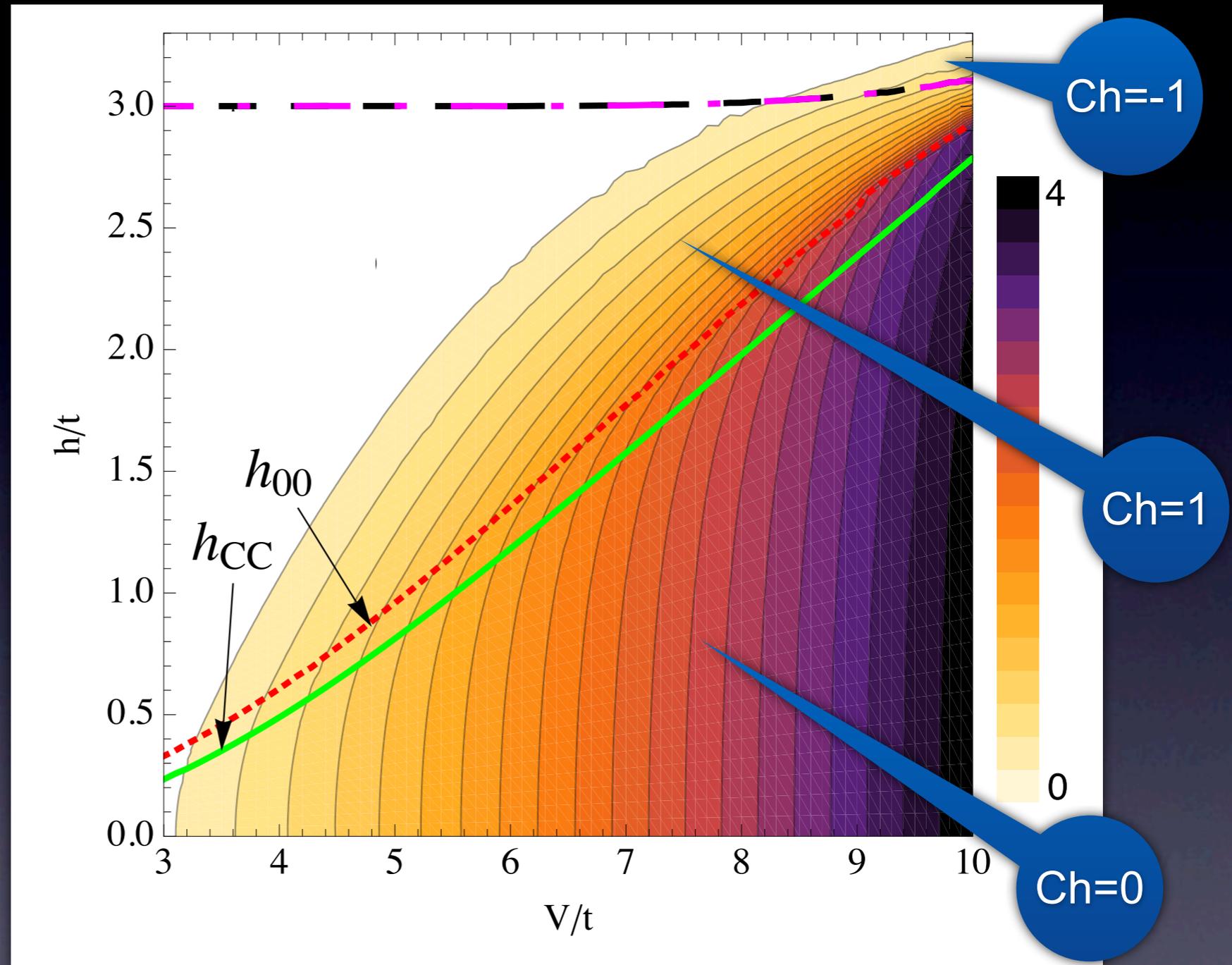
A. Kubasiak, P.M. & M. Lewenstein, EPL 2010

# Spin imbalance **vs.** pair breaking

without SO coupling:  
analytic CC limit  
(  $h_{CC} = \Delta_0/\sqrt{2}$  )

with SO coupling:  
self-consistent calculation of  $\Delta$   
from the BCS gap equation

$$\alpha = \beta = \pi/4 \quad \mu = -3t$$



A. Kubasiak, PM & M. Lewenstein, EPL 2010

# Synthetic dimensions

# Q. Sim. & Extra Dimensions

Quantum simulation with ultracold atoms:

- Hubbard model (SF-MI transition, ...)
- synthetic gauge fields (relativistic dispersions, ...)
- strongly-correlated states (QH, spin liquids, ...)

Extra (=non-spatial) dimensions:

- attempts to unify gravitation with electro-weak forces (Kaluza-Klein, Yang-Mills, ...)
- thermal QFT: compactification of euclidean time leads to Matsubara frequencies

(extra-dim is usually discrete and compact)

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quantum simulation of an extra dimension?

# The main idea

- use a system with  $D$  spatial dimensions
- encode the  $(D+1)^{\text{th}}$  dimension in a different degree of freedom (e.g., the spin)

$$H = -J \sum_{d=1}^{D+1} \sum_{\tilde{\mathbf{r}}} \hat{a}_{\tilde{\mathbf{r}}+\mathbf{u}_d}^\dagger \hat{a}_{\tilde{\mathbf{r}}} + \text{h.c.} \quad \tilde{\mathbf{r}} = (\mathbf{r}, \sigma)$$

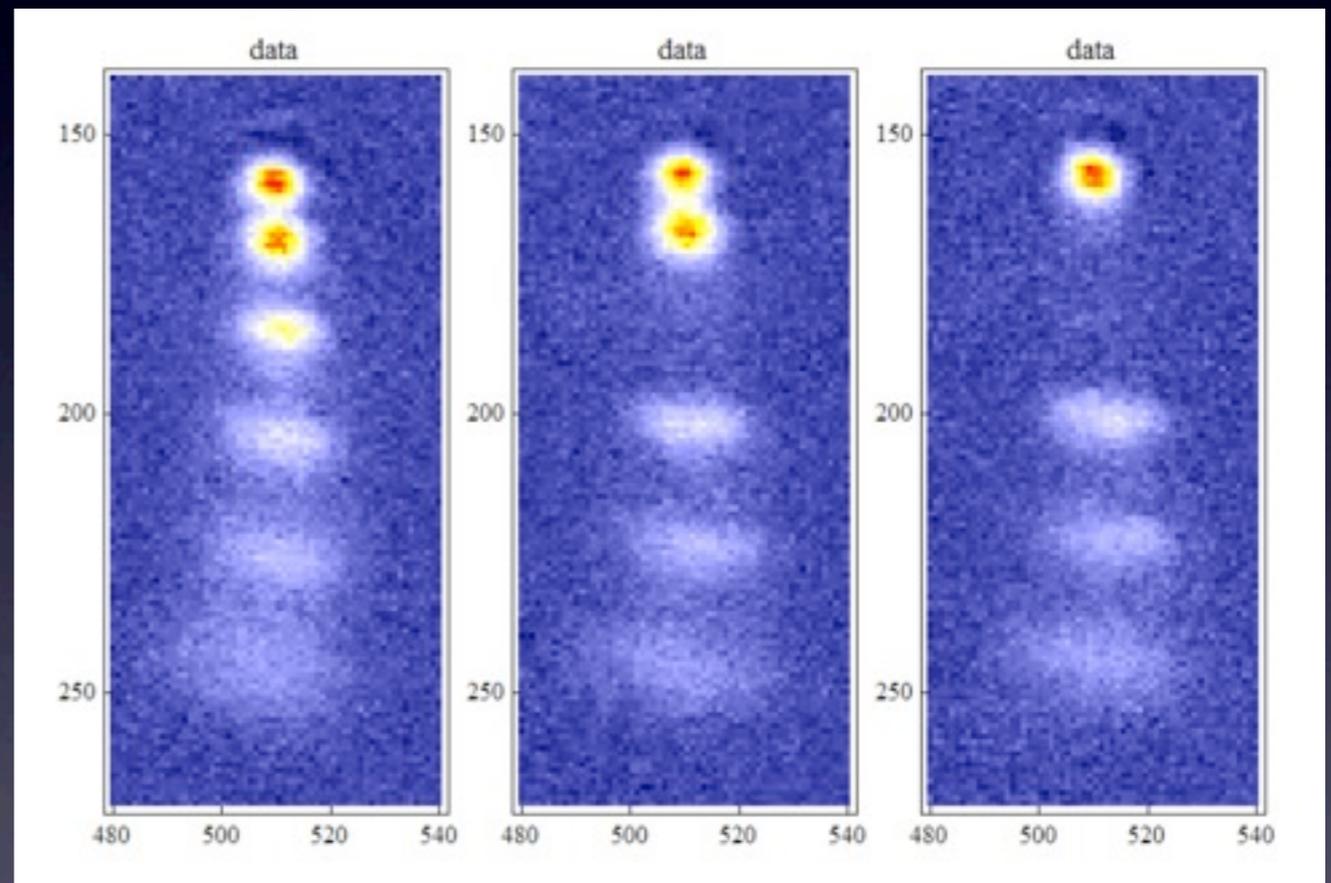
$$= -J \sum_{\sigma=1}^N \left[ \sum_{d=1}^D \sum_{\mathbf{r}} \hat{a}_{\mathbf{r}+\mathbf{u}_d}^{(\sigma)\dagger} \hat{a}_{\mathbf{r}}^{(\sigma)} + \hat{a}_{\mathbf{r}}^{(\sigma+1)\dagger} \hat{a}_{\mathbf{r}}^{(\sigma)} \right] + \text{h.c.}$$

**important:** allow only nearest-neighbor “spin-tunneling”

# Large N systems

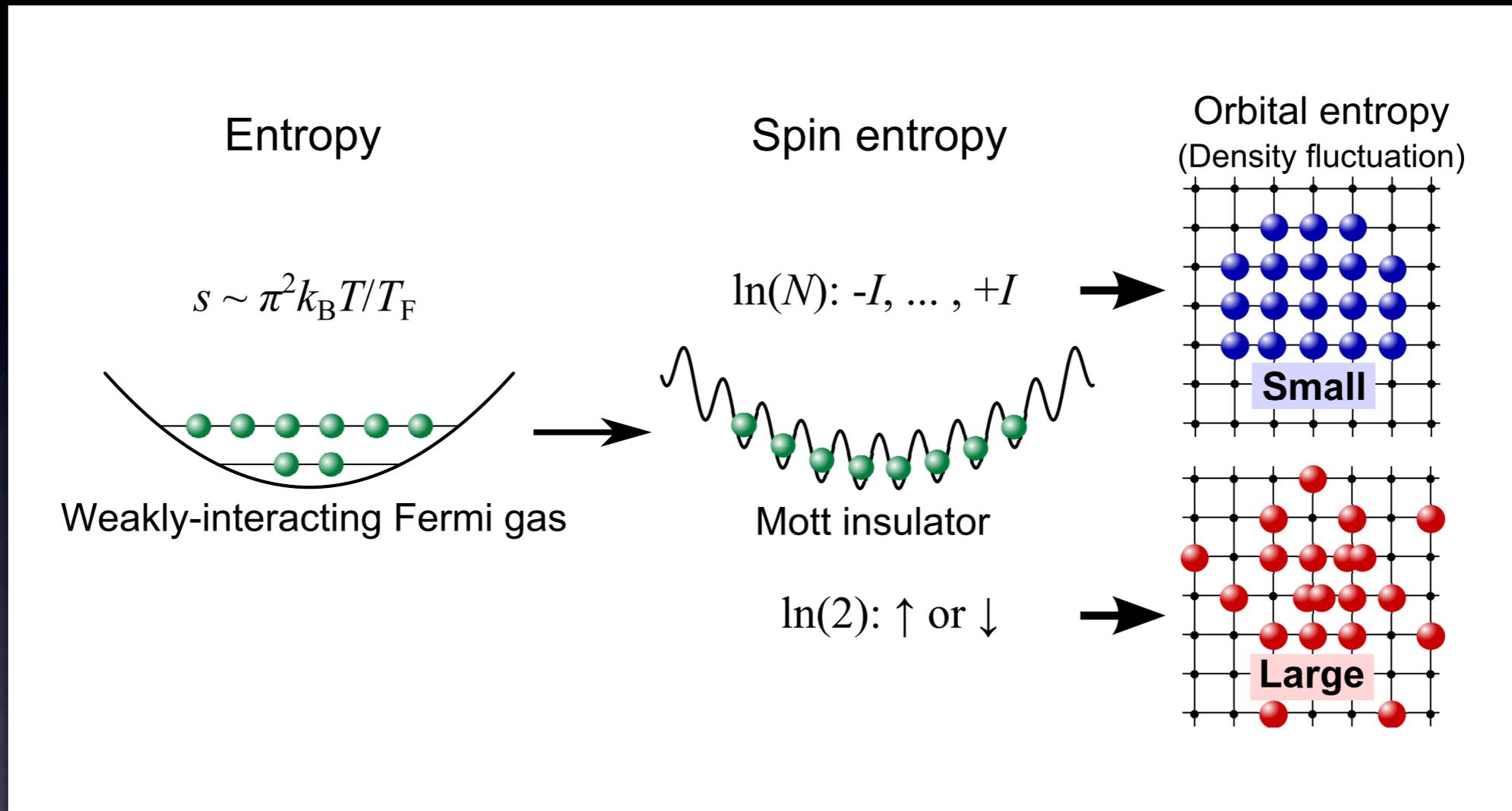
species	N
Li	2,3,...
$^{87}\text{Rb}$	3
$^{173}\text{Yb}$	6
$^{40}\text{K}$	2,...,10
$^{87}\text{Sr}$	10
$^{165}\text{Ho}$	120

$^{173}\text{Yb}$  at LENS:



interactions in earth-alkali atoms are  $\text{SU}(N)$  invariant!

# SU(6) Mott insulator

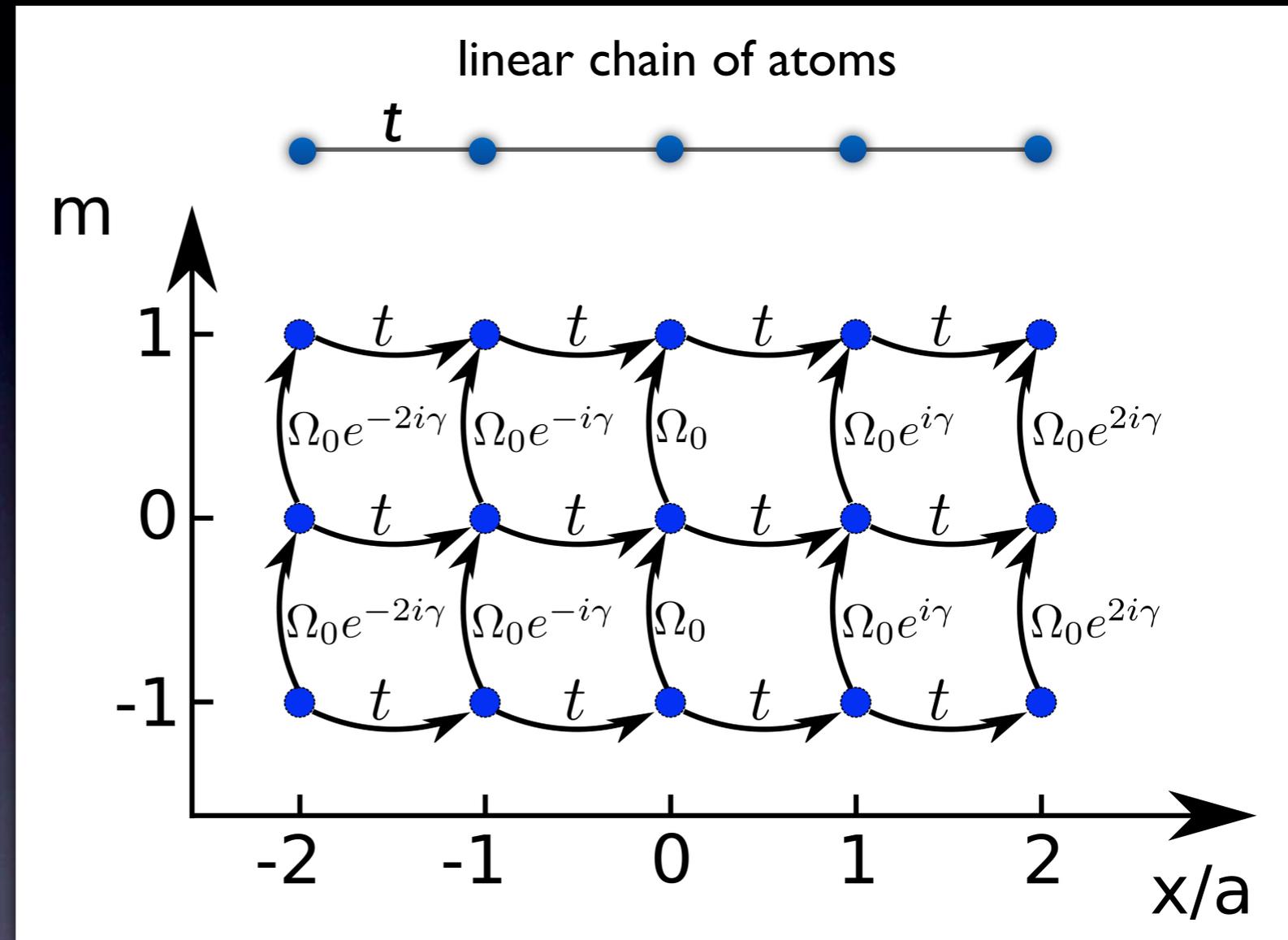
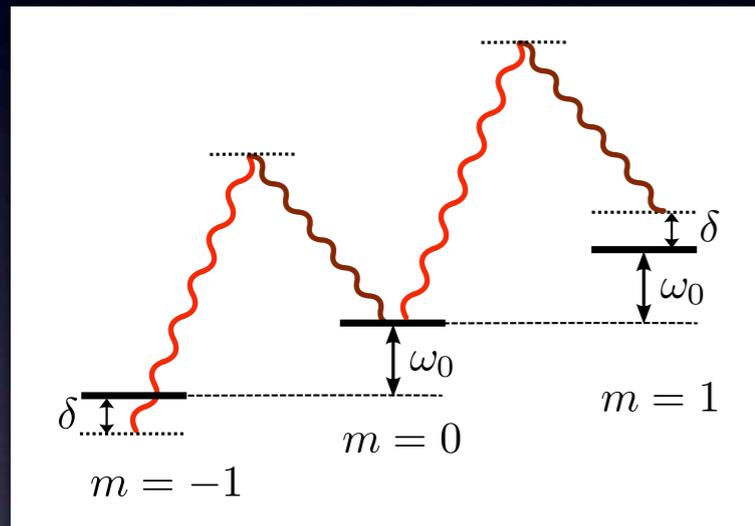


S. Taie, R. Yamazaki, S. Sugawa, and Y. Takahashi, Nature Phys. 2012

Novel cooling mechanisms?

# Implementation

laser-assisted  
spin-tunneling



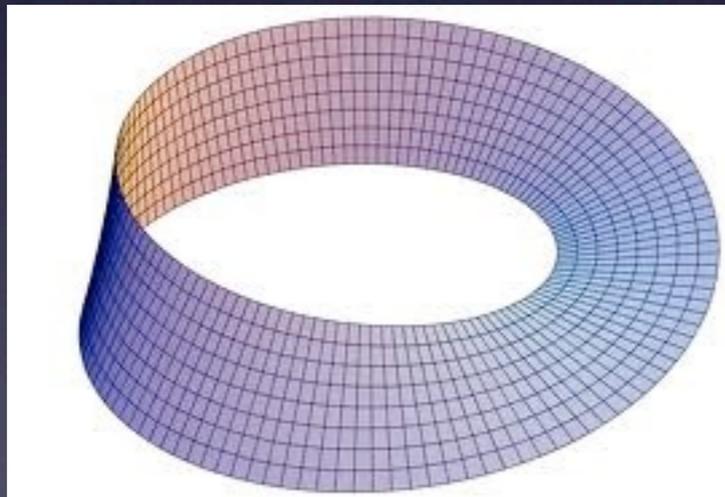
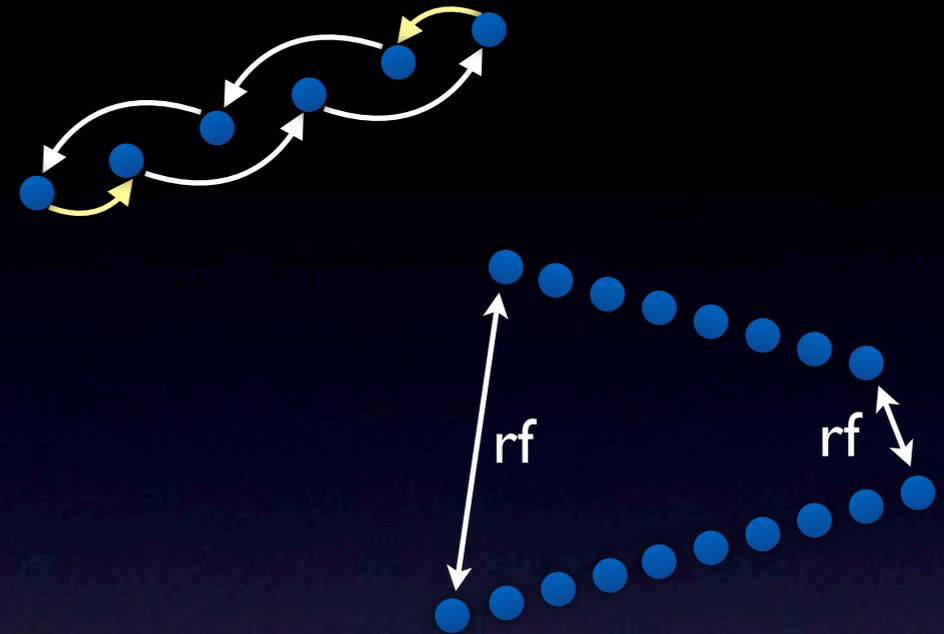
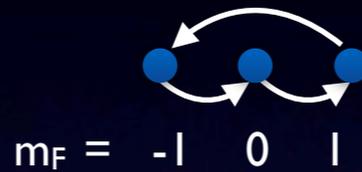
yields strong and non-staggered magnetic fluxes  
and long-ranged interactions

PM, A. Celi, I. Spielman, G. Juzeliunas, and M. Lewenstein, in preparation

# Interesting topologies

possible boundary conditions along the spin direction:

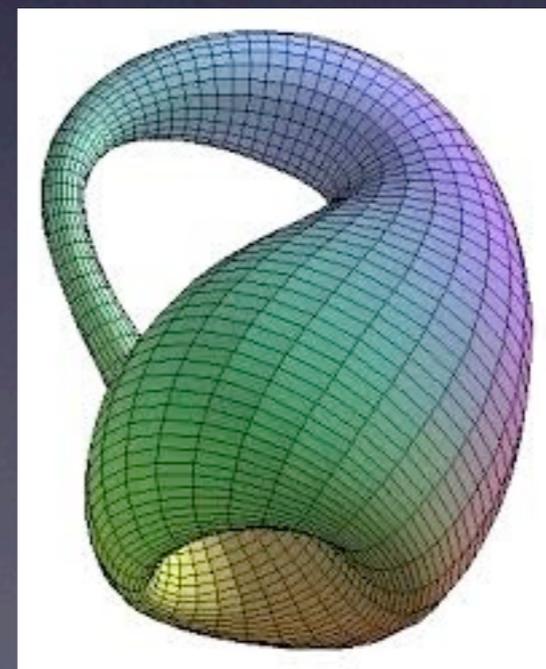
- open
- closed
- twisted (a closed loop encircles a phase)



**Möbius strip**  
linear chain in the spatial dir.,  
 $\pi/2$  twist in spin

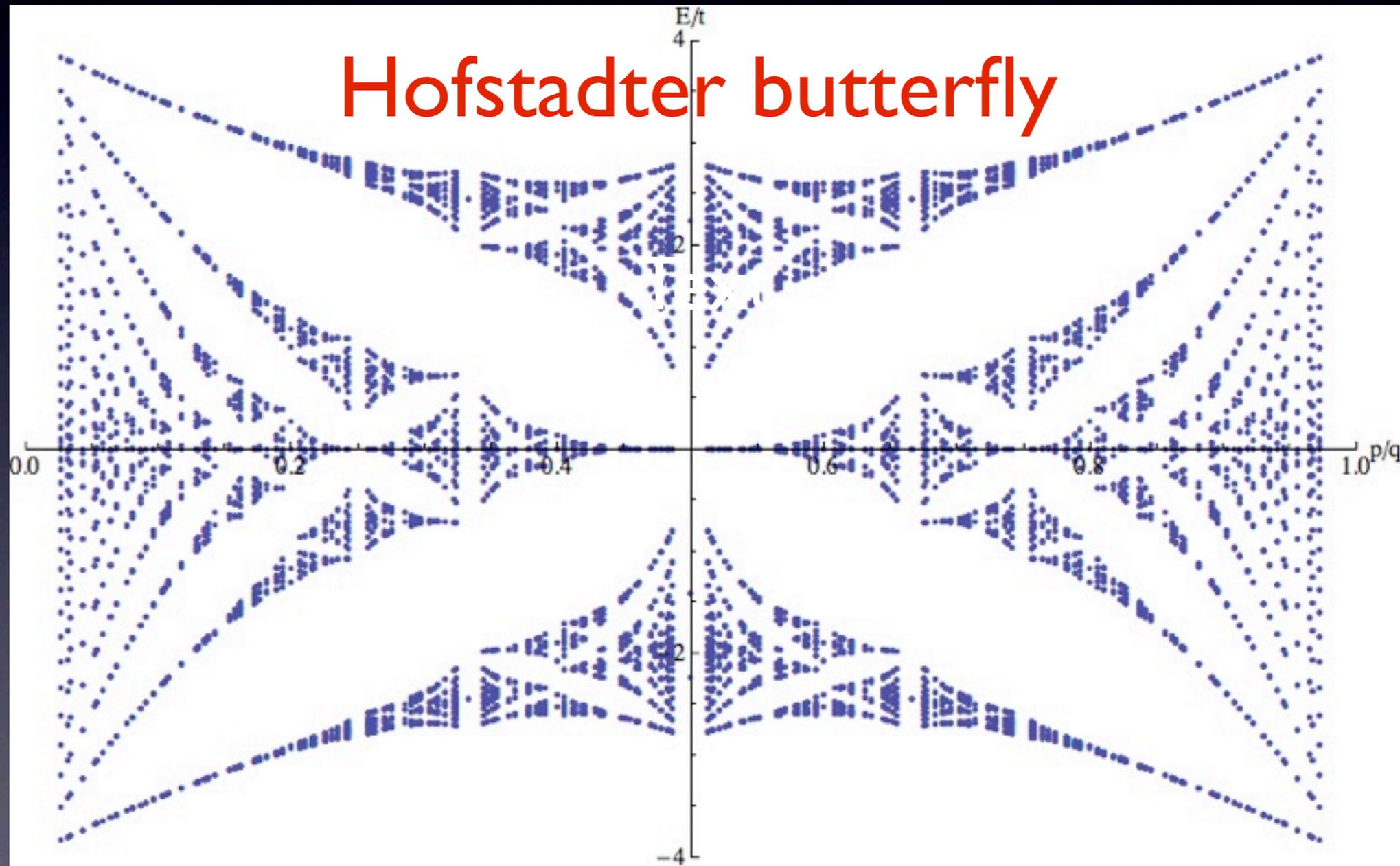
## Klein bottle

ring in the spatial dir.,  
 $\pi/2$  twist in spin



# Energy spectrum of spin-1 atoms with closed b.c. and non-zero flux

$$\Phi = 2\pi(p/q)$$



# in collaboration with



Maciej Lewenstein



Alessio Celi



Gediminas Juzeliunas



Ian Spielman



Anna Kubasiak



Anna Sanpera

# Conclusions

- Gauge fields yield non-trivial topological phases
- Superfluidity in  $\uparrow\downarrow$  fermions is stabilized by a non-Abelian gauge field
- Using an internal d.o.f. as an extra-dimension:
  - ★ quantum simulation of high-energy theories, and  $D>3$  systems (e.g., crit. exp. of ph. trans.)
  - ★ novel cooling schemes possible?

PM, A. Sanpera & M. Lewenstein, PRA(R) 2010

A. Kubasiak, PM & M. Lewenstein, EPL 2010

O. Boada, A. Celi, J.I. Latorre, and M. Lewenstein, PRL 2012

PM, A. Celi, I. Spielman, G. Juzeliunas, and M. Lewenstein, in preparation