Efimov trimers under strong confinement

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Introduction

The dimensionality of a system fundamentally affects the behaviour of interacting quantum particles. Classic examples range from the fractional quantum Hall effect to high temperature superconductivity. In general, one expects confinement to favour the binding of particles.

However, attractively interacting bosons defy this expectation: while three identical bosons in 3D can support an infinite tower of Efimov trimers, only two universal trimers exist in the 2D case.

Here we reveal how these two limits are connected, by investigating the problem of *three identical bosons confined by a harmonic potential along one direction*.

Our results suggest a way to use strong confinement to engineer more stable Efimov-like trimers, which have so far proved elusive.

Bound states in 3D and 2D

3D:

- for a>0, one dimer with energy $-1/ma^2$
- ∞ trimers, whose energies may mapped onto another via the transformations $E \to \lambda_0^{-2n} E$ and $a \to \lambda_0^n a$ [1,2]

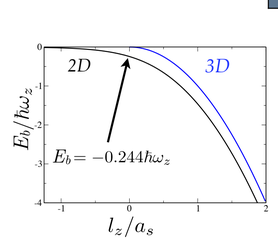
$$V(z) = \frac{1}{2} m\omega_z^2 z^2$$

2D:

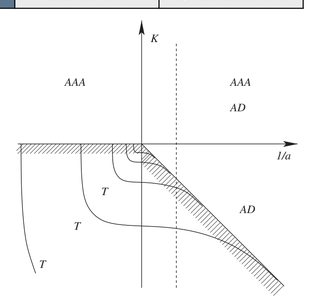
- only one dimer of energy E_b, for any scattering length
- two trimers, of energies $-16.5|E_b|$ and $-1.27|E_b|$ [3]

Scaling symmetry:

	3D	2D
2-body	Continuous	Continuous
3-body	Discrete	Continuous



$l_z = \sqrt{1/m\omega_z}$



Key findings

- confinement breaks the discrete Efimov scaling symmetry, and destroys the weakest bound trimers
- 3D and 2D trimers have the same symmetry (s-wave), so they can hybridize under confinement
- in the dimensional crossover, trimers tunnel through a short-range repulsive barrier
- the two deepest quasi-2D trimers are stabilized for arbitrarily weak attraction and confinement strength
- long-ranged quasi-2D trimers have small overlap with deep bound states: stable against three-body losses?

Three identical bosons in quasi-2D

When $T \ll \omega_z$ particles occupy the ground h.o. level, and are kinematically 2D. However, collisions are still 3D, and allow to populate virtually all h.o. excited states.

STM equation for strong confinement along z [4,5]:

$$\mathcal{T}^{-1}(\mathbf{k}_1, E_3 - \epsilon_{\mathbf{k}_1} - N_1\omega_\perp)\,\chi_{\mathbf{k}_1}^{N_1} = 2\sum_{\mathbf{k}_2, N_2, n_{23}, n_{31}} \frac{f_{n_{23}} f_{n_{31}} \langle N_1 n_{23} | N_2 n_{31}\rangle e^{-(k_1^2+k_2^2)/\Lambda^2} \chi_{\mathbf{k}_2}^{N_2}}{E_3 - \epsilon_{\mathbf{k}_1} - \epsilon_{\mathbf{k}_2} - \epsilon_{\mathbf{k}_1+\mathbf{k}_2} - (N_1 + n_{23})\omega_\perp}$$

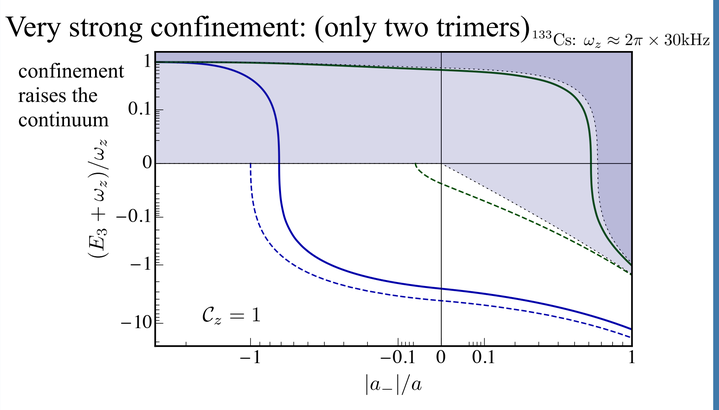
where:

- $\mathcal{T}(\mathbf{k}, E) = \frac{2\sqrt{2\pi}}{m}\left\{\frac{l_z}{a} - \mathcal{F}\left(\frac{-E + k^2/4m}{\omega_\perp}\right)\right\}^{-1}$
- E_3 is the energy measured from the 3-atom continuum
- $\mathbf{k}_i$ is the relative momentum of atom i w.r.t. the pair (j,k)
- n_{ij} and N_i are the h.o. quantum numbers for motion along z of a pair, and of an atom and a pair
- Λ is a UV cut-off fixing a_- (the scattering length at which the ground trimer crosses the 3-atom continuum)
- f_{n_r} is the wave function of the relative motion at z_r=0

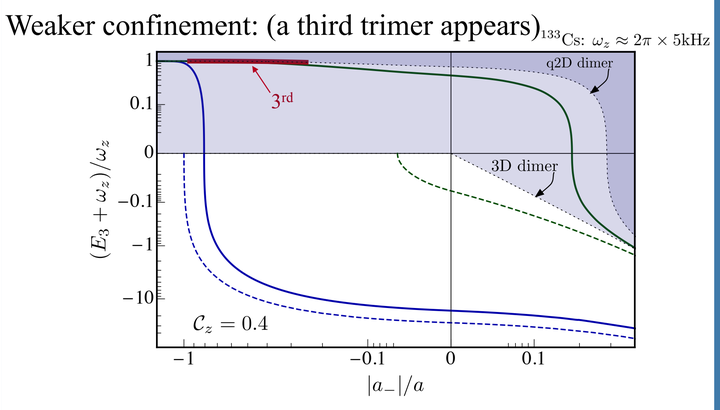
Spectra (3D style)

Confinement strength: $\mathcal{C}_z \equiv |a_-|/l_z$

Very strong confinement: (only two trimers) ^{133}Cs: $\omega_z \approx 2\pi \times 30$kHz



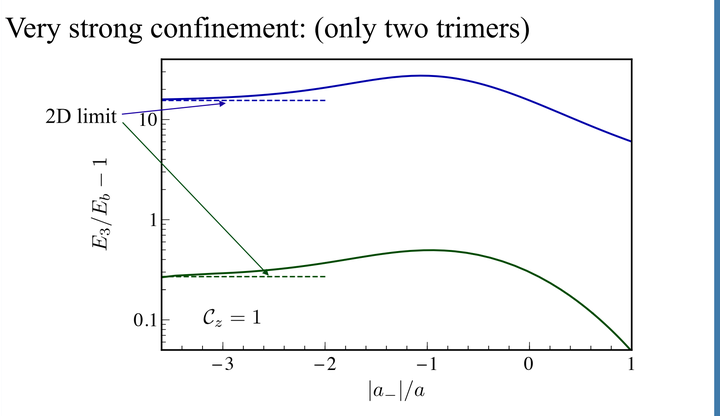
Weaker confinement: (a third trimer appears) ^{133}Cs: $\omega_z \approx 2\pi \times 5$kHz



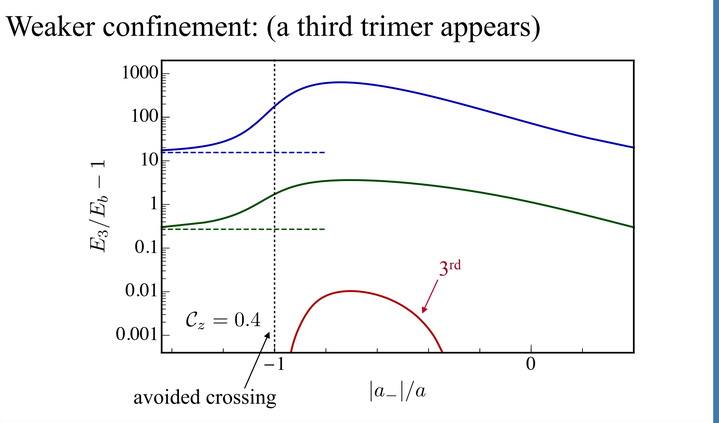
Trimer energy $E \sim \omega_z$ even outside its regime of existence in 3D → resistant to thermal dissociation when $T \ll \omega_z$

Spectra (2D style)

Very strong confinement: (only two trimers)



Weaker confinement: (a third trimer appears)



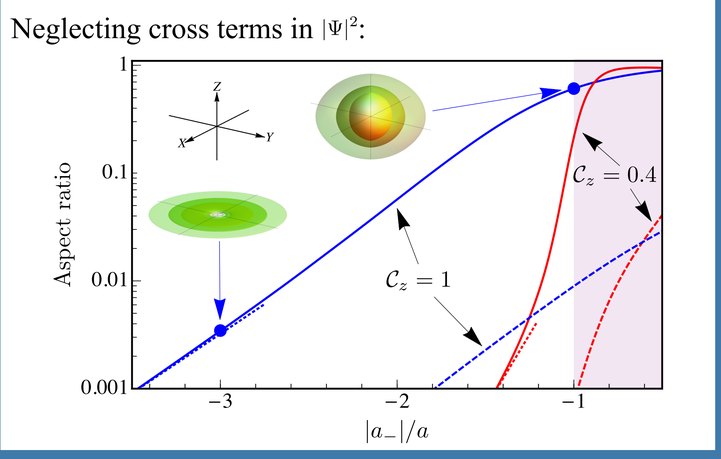
Aspect ratios $2\langle Z^2\rangle / \langle \rho^2\rangle$

Fadeev decomposition:

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \psi^{(1)}(\mathbf{r}_{23}, \mathbf{r}_{1,23}) + \psi^{(2)}(\mathbf{r}_{13}, \mathbf{r}_{2,31}) + \psi^{(3)}(\mathbf{r}_{12}, \mathbf{r}_{3,12})$$

$$\psi^{(1)}_{\mathbf{k}_{23}, \mathbf{k}_1, n_{23}, N_1} \propto \frac{f_{n_{23}} \chi_{\mathbf{k}_1}^{N_1}}{E_3 - k_{23}^2/m - 3k_1^2/4m - (n_{23} + N_1)\omega_\perp}$$

Neglecting cross terms in $|\Psi|^2$:



Hyper-spherical potentials and wave functions

Hyper-spherical expansion: $R^2 = r_1^2 + r_2^2 + r_3^2$

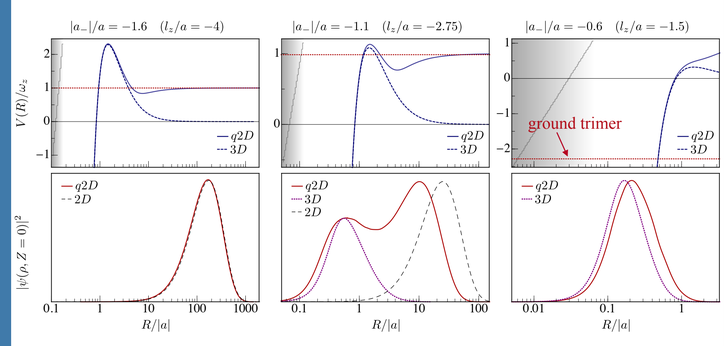
$$\Psi(R, \Omega) = \frac{1}{R^{5/2}\sin(2\alpha_k)} \sum_{n=0}^{\infty} f_n(R)\Phi_n(R, \Omega)$$

Hyper-radial Schrödinger equation:

$$\left[-\frac{1}{2m}\frac{\partial^2}{\partial R^2} + V(R)\right] f_0(R) = (E_3 + \omega_z)\, f_0(R)$$

$V(R)$ depends on l_z/a but not on the 3-body parameter.

When $l_z/a < -1$, the potential shows a barrier at $R \approx |a|$ with height $\sim 0.15/ma^2$



Wave function for the atom-pair relative motion:

$$\psi(\boldsymbol{\rho}, Z) = R^{3/2} \sum_{\mathbf{k}, N} e^{i\mathbf{k}\cdot\boldsymbol{\rho}} \phi_N(Z) \chi_{\mathbf{k}}^N$$

Crossover trimers have very small weight at short distances, i.e., small overlap with deeply bound states.

Experimental consequences

- the deepest trimer is rather 3D-like for realistic trappings
- no trimer resonances observable when ω_z is larger than the repulsive barrier arising from 3D physics; as such, the loss peak of the ground trimer disappears for $\mathcal{C}_z \gtrsim 0.4$
- due to universality of the Efimov spectrum, the loss peak of the first excited trimer disappears for $\mathcal{C}_z \gtrsim 0.4/22.7$

^{133}Cs: $\omega_z \simeq 2\pi \times 9$Hz

Outlook

- similar avoided crossings should appear in the spectrum of quasi-1D trimers
- since exactly two tetramers are predicted in 2D, we expect those to resist in the dimensional crossover, and actually be stabilized for arbitrarily weak z-confinements
- strong confinement may be used to to engineer more stable, Efimov-like, hybrid trimers: the small weight of the trimer wave function at short distances, due to the presence of a repulsive barrier, should allow for the formation of a many-body state of long-lived trimers.

References

[1] V. Efimov, *Energy levels arising from resonant two-body forces in a three-body system*, Phys. Lett. B **33**, 563 (1970).

[2] E. Braaten and H.-W. Hammer, *Universality in few-body systems with large scattering length*, Phys. Rep. **428**, 259 (2006).

[3] L. W. Bruch and J. A. Tjon, *Binding of three identical bosons in two dimensions*, PRA **19**, 425 (1979).

[4] J. Levinsen, T. G. Tiecke, J. T. M. Walraven, and D. S. Petrov, *Atom-Dimer Scattering and Long-Lived Trimers in Fermionic Mixtures*, PRL **103**, 153202 (2009); J. Levinsen and M. M. Parish, *Bound States in a Quasi-Two-Dimensional Fermi Gas*, PRL **110**, 055304 (2013).

[5] J. Levinsen, P. Massignan, and M. M. Parish, *Efimov trimers under strong confinement*, Phys. Rev. X 4, 031020 (2014).

Acknowledgments

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