# Nondiffractive light in photonic crystals

- Nondiffractive light in linear Photonic Crystals (PCs)
  - Nondiffractive beams in PCs;
  - Nondiffractive pulses in PCs;
  - Nondiffractive resonators of PCs;
- Nondiffractive light in nonlinear PCs
  - Subdiffractive solitons in nonlinear PCs;
  - Nondiffractive nonlinear resonators

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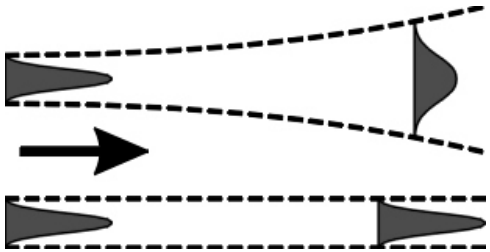
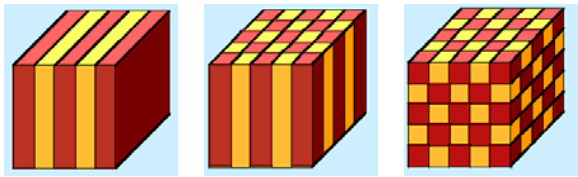
Group of "Nondiffractive light" : K.Staliunas, R.Herrero, C.Serrat, C.Cojocaru, J.Trull.

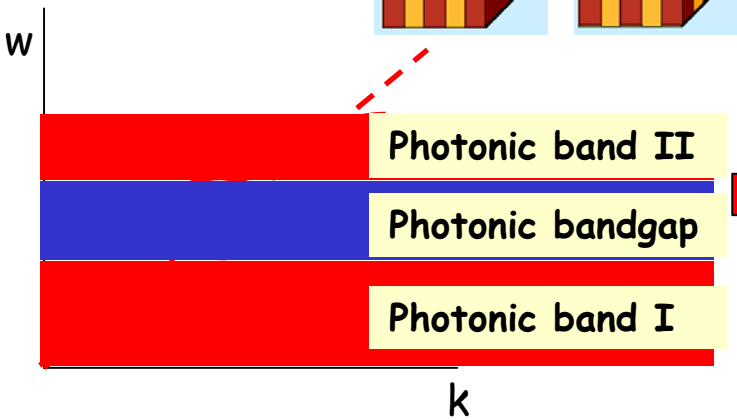


# Nondiffractive light in linear Photonic Crystals

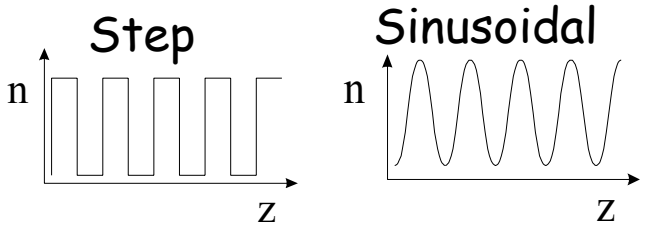
## Photonic Crystals

1D,2D,3D:






## Configurations:




- Photonic bands: conductors and insulators of light
- Modification of dispersion: slow light
- Modification of diffraction

•diffraction management in fiber arrays (1D PCs)

•diffraction management in BECs (1D lattices)

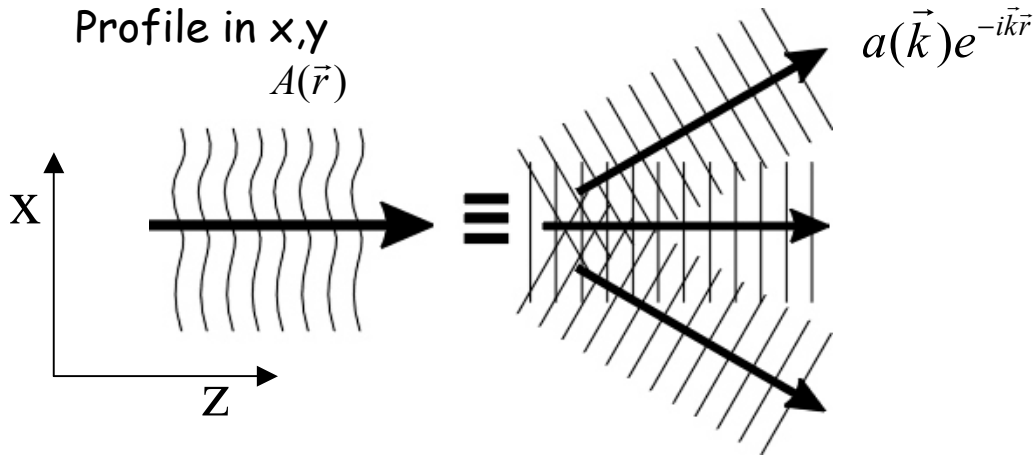
•diffraction management, numerics and experiments in 2D PCs

But also **Elimination of diffraction !!**

# Diffraction

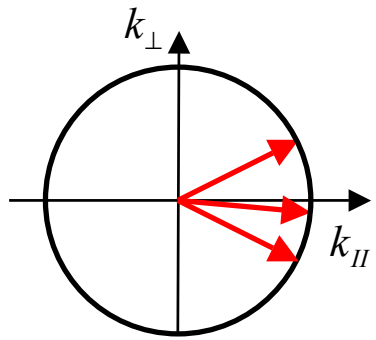
Beam with carrier frequency $\omega_0$ propagating in z direction :

$$E(\vec{r}) = A(\vec{r})e^{i\omega_0 t}$$




$$\vec{k} = (k_\perp, k_\parallel)$$

$$k = \sqrt{k_\parallel^2 + k_\perp^2} = k_0 = \frac{\omega_0}{c}$$

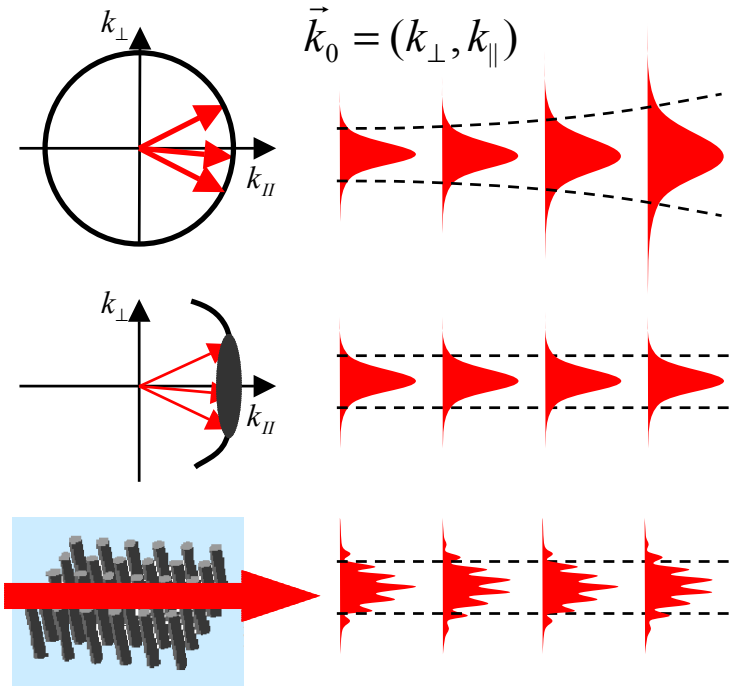


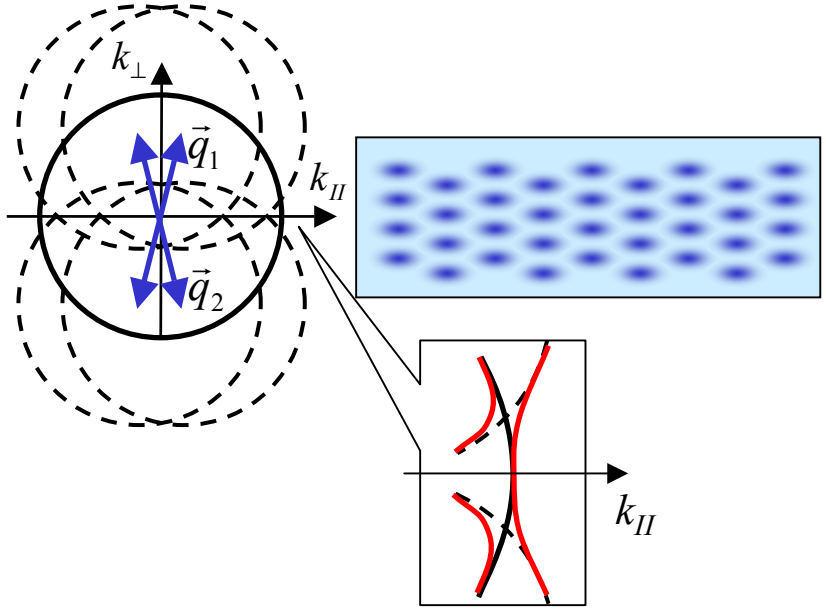

$$A(\vec{r}) = \int_{-\infty}^{\infty} a(k_\perp)e^{-ik_\perp x}e^{-ik_\parallel z}dk_\perp \quad , \quad k_\parallel = \sqrt{k_0^2 - k_\perp^2}$$

**Different $k_{||}$ for each plane wave** → **Relative phase variations during the propagation in z** → **Variation of the interference pattern in z**

**Diffraction**

# Diffraction management



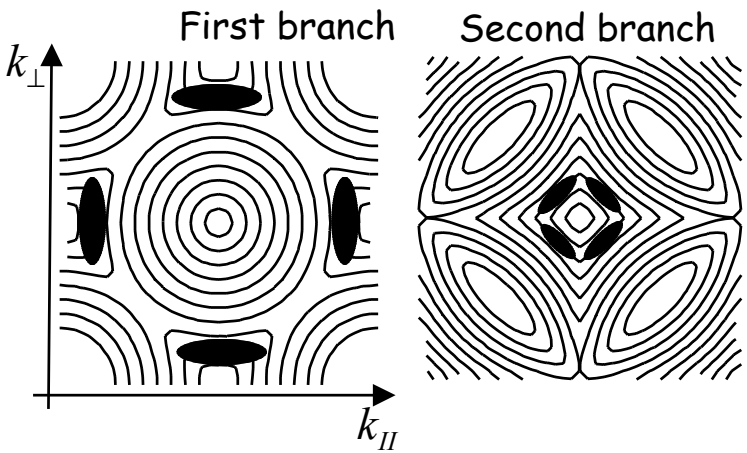




•Localization of non-diffractive regimes

$$\nabla \times \left[ \frac{1}{\varepsilon(\vec{r})} \nabla \times H(\vec{r}) \right] = \left( \frac{\omega}{c} \right)^2 H(\vec{r})$$

H.Kosaka e.a. 1999, J.Witzens e.a. 2002,
D.N.Chigrin e.a. 2003, R.Iliew e.a. 2004,...

Constant ω surf. First brillouin zones



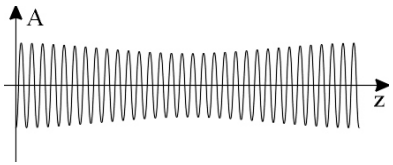
# Slowly Varying Envelope Approximation

- Equació d'ones per l'ona electro-magnètica:

$$\nabla^2 \mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \qquad \mathbf{E}(\vec{r}) = E(\vec{r})e^{-i\omega_0 t} = A(\vec{r})e^{i(k_0 z - \omega_0 t)}$$

- Si considerem que $\frac{\partial E}{\partial t} = 0$ : $\nabla^2 E + \frac{n^2\omega_0^2}{c^2}E = 0$

- Si considerem que $\frac{\partial^2 A}{\partial z^2} << 0$




: $\nabla_\perp^2 A + 2ik_0\frac{\partial A}{\partial z} = 0$

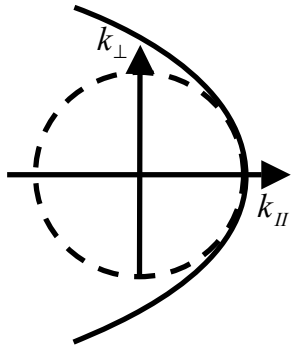
- Sols considerant una variable transversa i afegint $\Delta$n:

$$\left(2ik_0\,\partial/\partial z + \partial^2/\partial x^2 + 2\Delta n(x,z)k_0^2\right)A(x,z) = 0$$

# Paraxial approx. and sinusoidal Δn

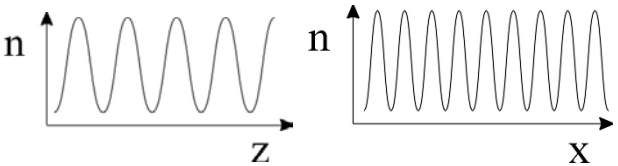
## Paraxial

$$\left(2ik_0\,\partial/\partial z+\partial^2/\partial x^2+2\Delta n(x,z)k_0^2\right)A(x,z)=0$$



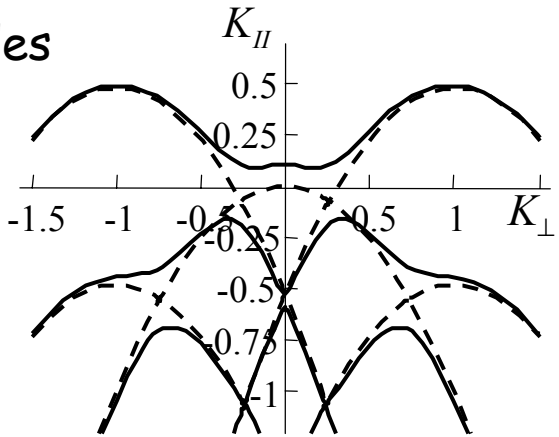

## Sinusoidal

$$\Delta n(\vec{r})=2m\left(\cos(\vec{q}_1\vec{r})+\cos(\vec{q}_2\vec{r})\right)$$




$$A(x,z)=\sum_{l,m}A_{l,m}e^{i\vec{k}_{l,m}\vec{r}}\quad \vec{k}_{l,m}=(k_\perp+lq_\perp,k_{II}+mq_{II})$$ harmonics

- Bloch modes



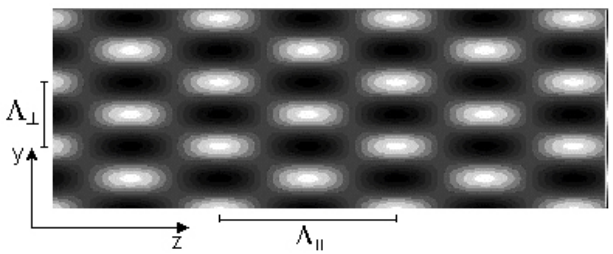

**Allows:**

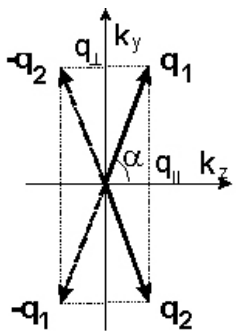
- Analytical expressions
- Generalizations:
  - Linear
  - Nonlinear (e.g. to BECs);

# Inscription of the index modulation

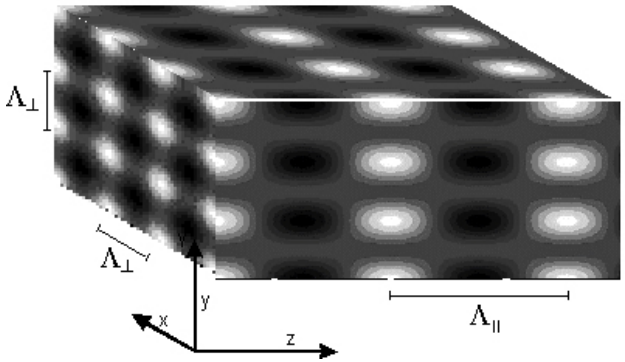
$$\Delta n(\vec{r}) = 2m\left(\cos(\vec{q}_1\vec{r}) + \cos(\vec{q}_2\vec{r})\right)$$

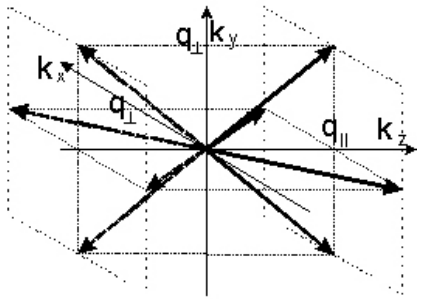
2D




3D




- <u>Photorefractive materials</u>

$$\Delta n(E)$$

Inscribing using light beams

- <u>Acusto-optic materials</u>

$$\Delta n(P)$$

Inscribing using ultrasound waves

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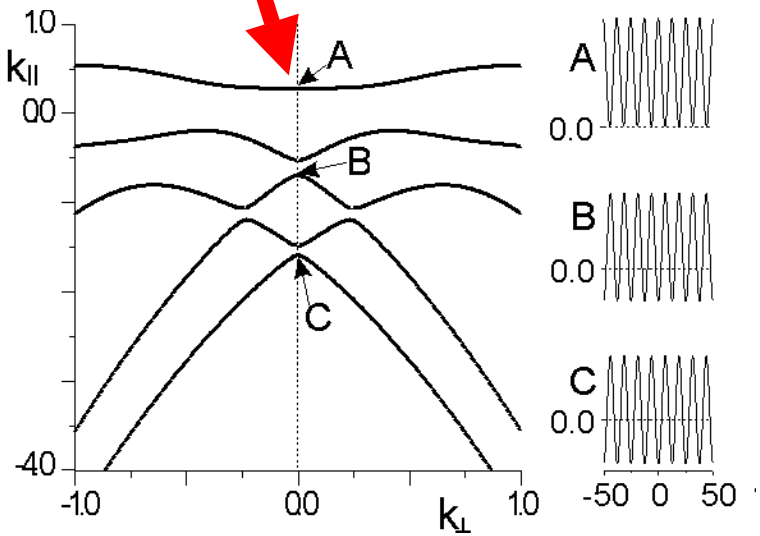
Dynamical photonic crystals → Dynamical diffraction management

# Diffraction elimination

Most homogeneous Bloch mode (A)

Zero diffraction:

$$\left.\frac{\partial^2 k_{\parallel}}{\partial k_{\perp}^2}\right|_{k_{\perp}=0} = 0$$

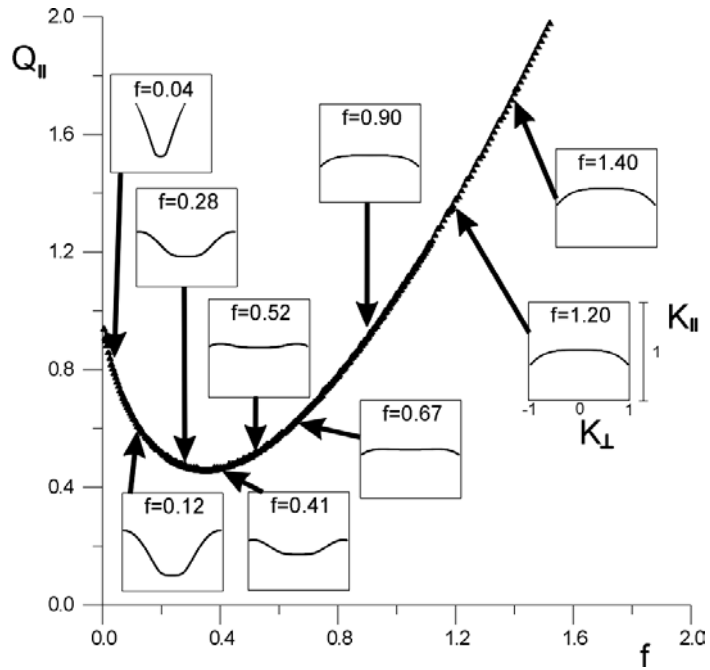



Normalization

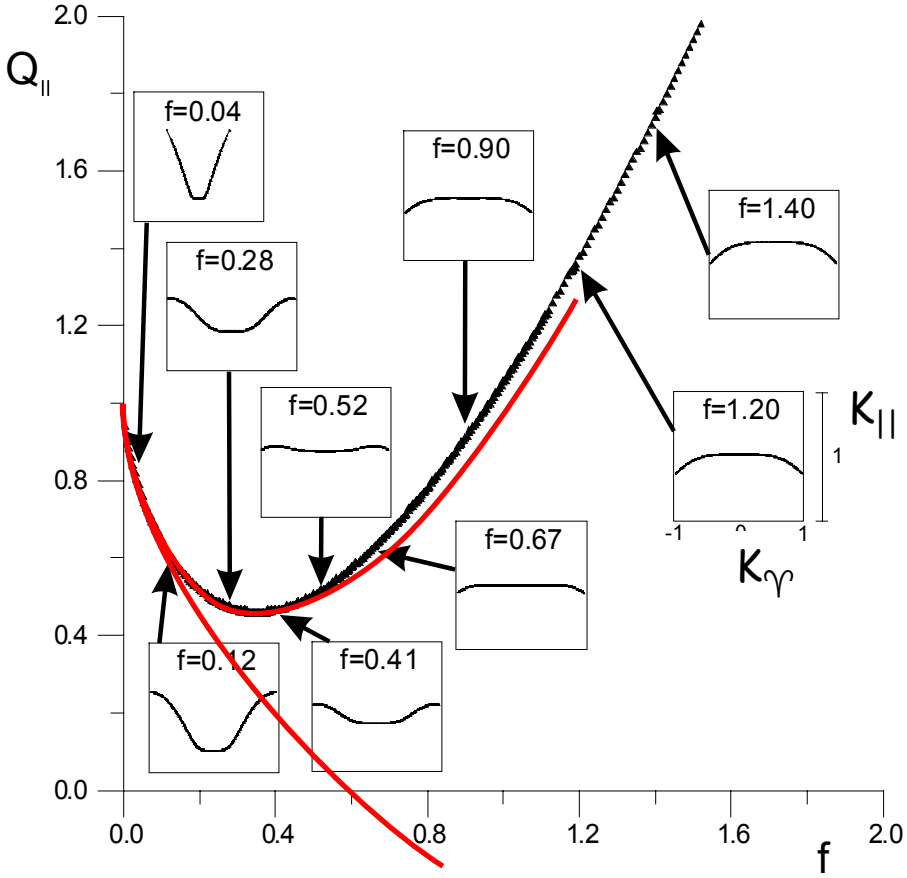
$f = 2mk_0^2 / q_{\perp}^2$ - modulation depth

$Q_{II} = 2q_{II}k_0 / q_{\perp}^2$ - geometry

Non-diffractive curve



# Analytical curve for f<<1




$f << 1:$ 3 relevant harmonics

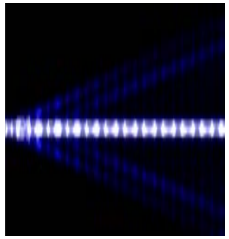
Transverse dispersion relation:
Power series expansion

$$d_2 \approx 1 - 8f^2/(1-Q_{II})^3 + 64f^4/(1-Q_{II})^5 + \ldots$$

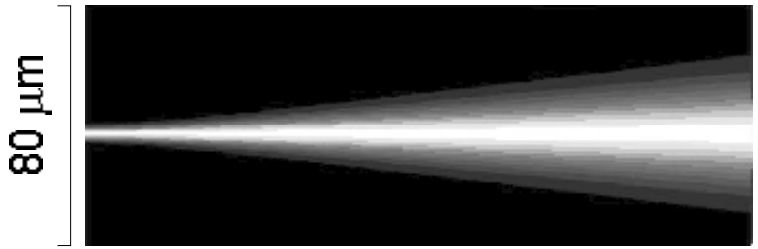
$$Q_{II}\big|_{d_2=0} \approx 1 - 2f^{2/3} + 4f^{4/3}/3 + \ldots$$

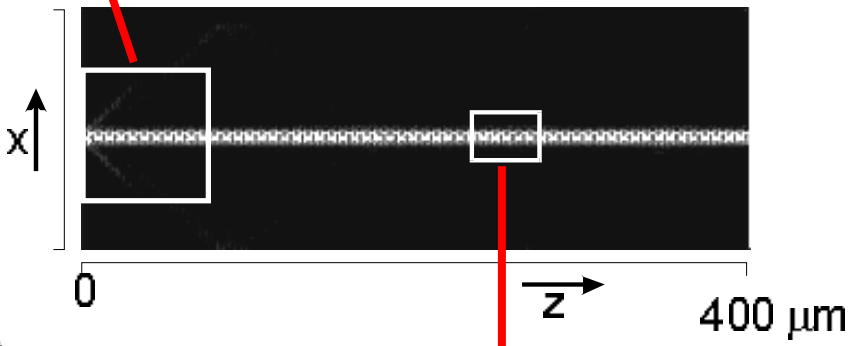
# Numerics 2D

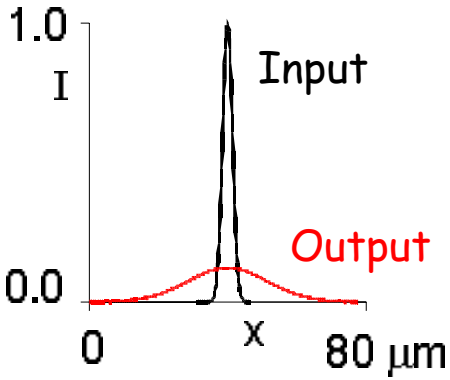
Homogeneous material
(diffractive)

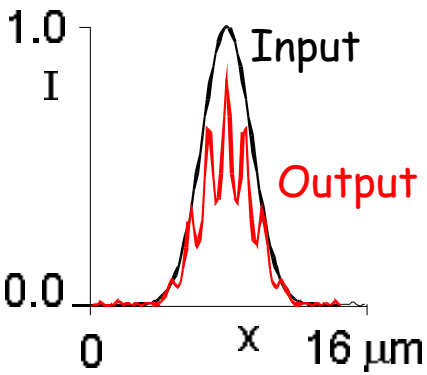


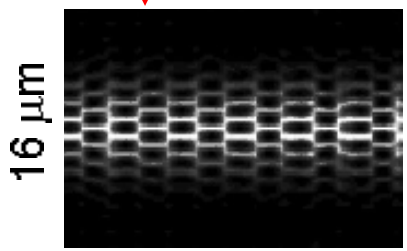
Photonic crystal
(non-diffractive)



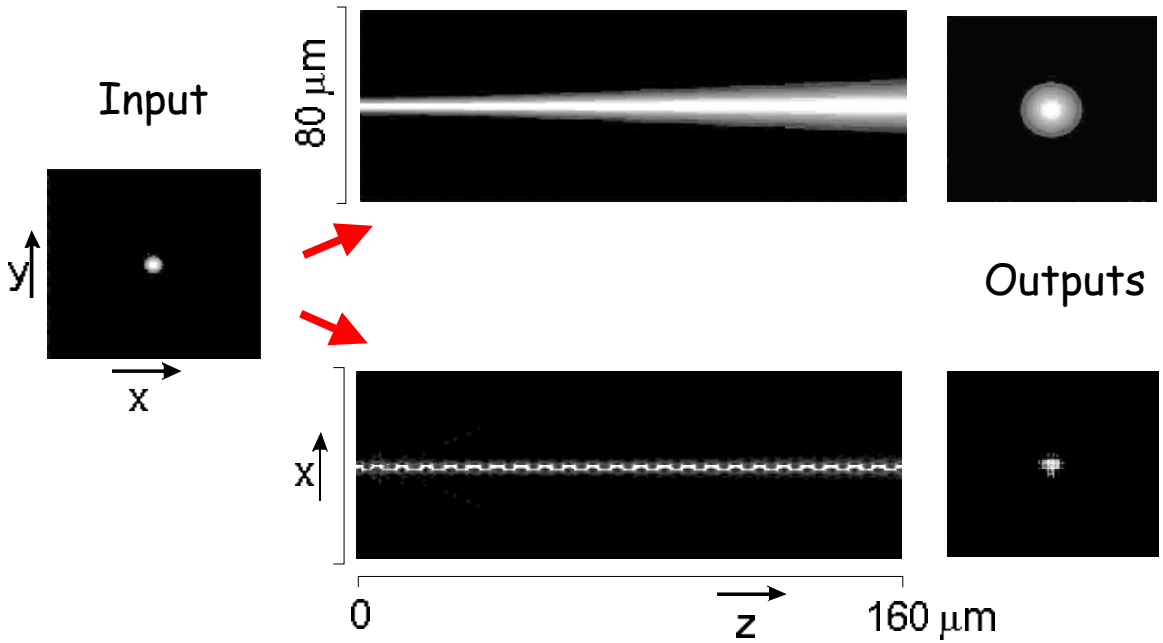



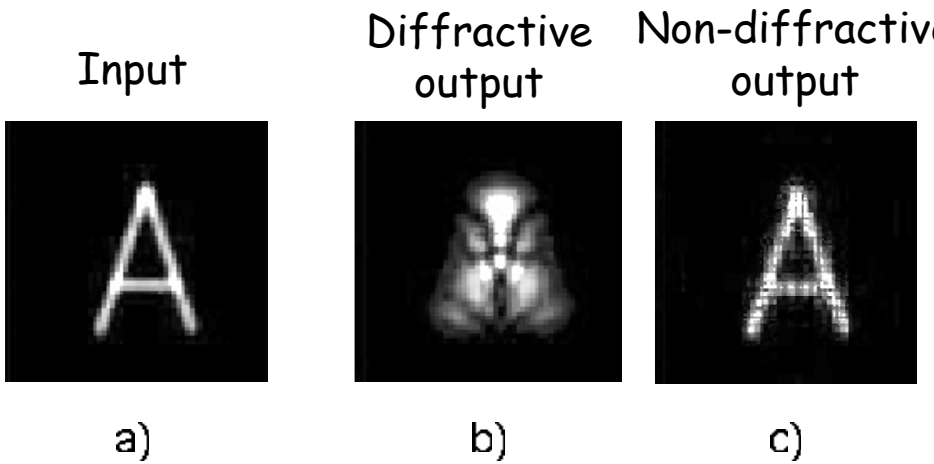
# Numerics 3D



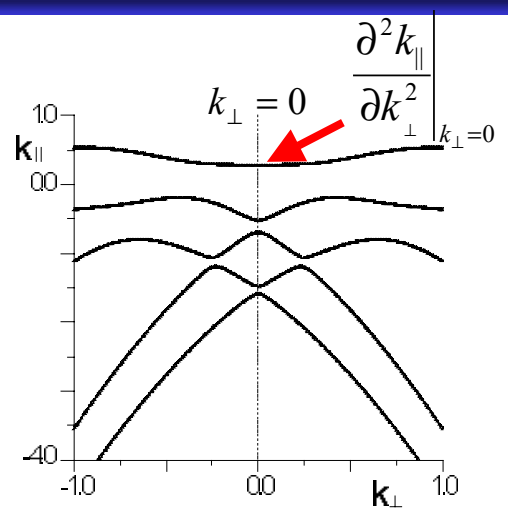

Homogeneous material (diffractive)

Photonic crystal (non-diffractive)

Linear effect



# Subdiffraction. Analytic




$$K_{II}(K_\perp) = \sum_{n=0}^{\infty} d_n K_\perp^n \qquad d_n = 1/n! \cdot \partial^n K_{II} / \partial K_\perp^n$$

n even

$$d_4 \approx 32 f^2 / (1 - Q_{II})^5 + \ldots$$

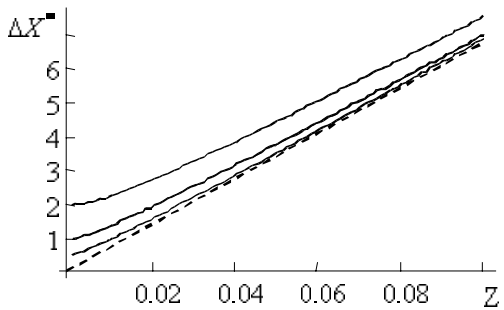
$$d_2 \to 0 \quad \Rightarrow \quad d_4\big|_{d_2=0} \approx f^{-4/3} + \ldots$$

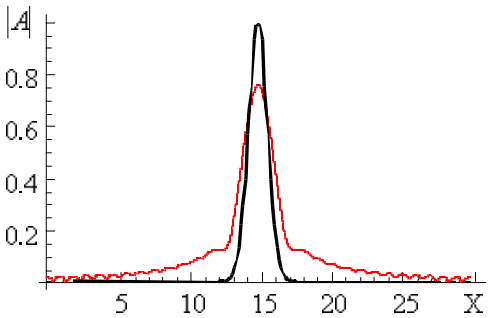
Bloch mode evolution: $\left(i\partial/\partial Z + d_4\, \partial^4/\partial X^4\right) A_B(X,Z) = 0$

Width evolution:

$$\Delta x^4(z) \approx 8 d_4 z / \left(\pi k_0 q_\perp^2\right)$$

$$\Delta x(z) = a \sqrt[4]{z}$$




**Example:** Beam width=10μm , λ=10μm

- **Homogeneous media**

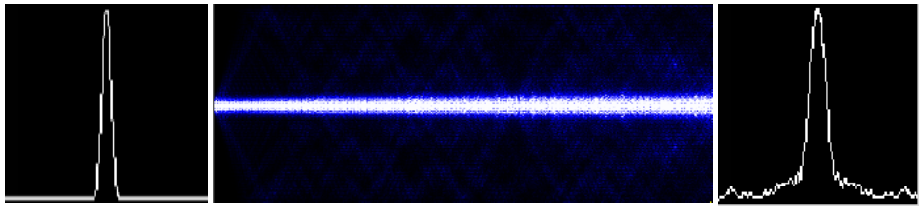
  Rayleigh length $z_0 = \pi \frac{W_0^2}{\lambda}$ ⏲ **30μm**

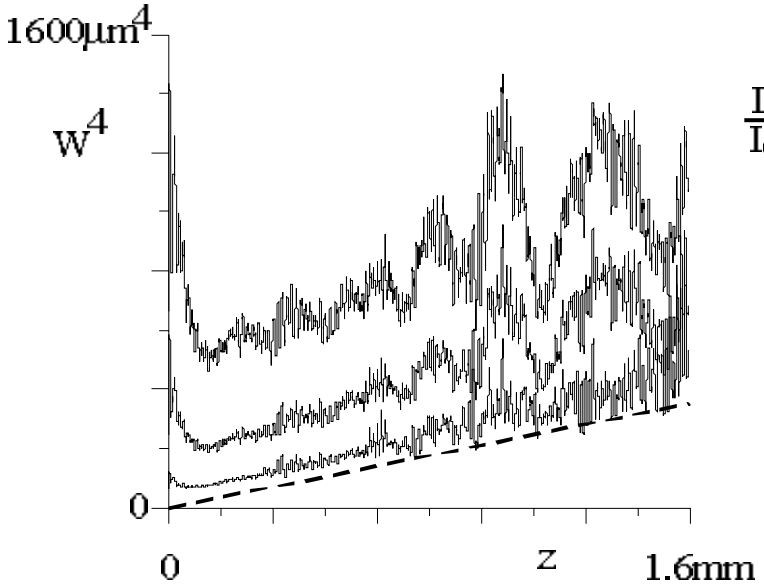
- **Non-diffractive PC**

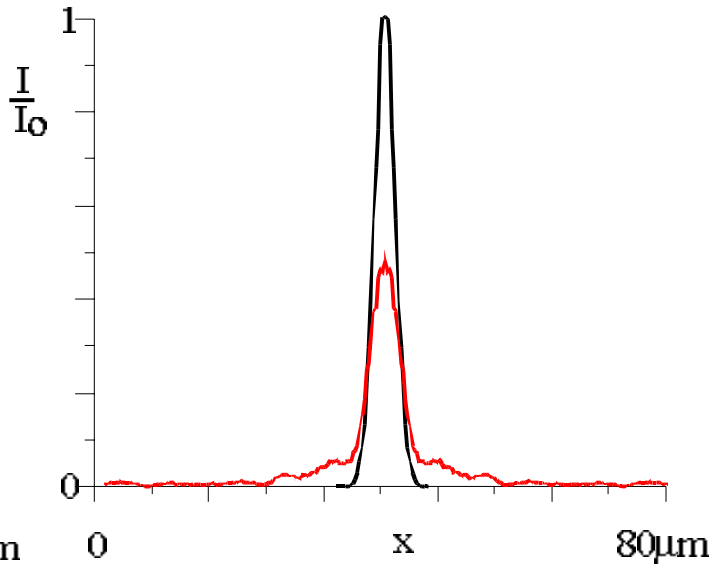
Nondiffractive propagation length ⏲ **2m**

# Subdiffraction. Numerics

Large propagation distance



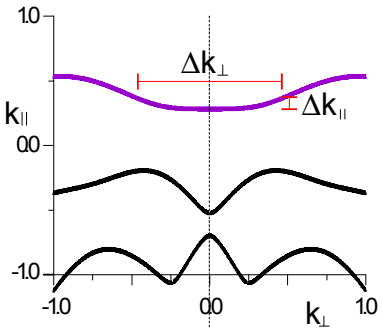


# The smallest non-diffractive structures

## Technical applications

Effective diffraction lower than a given value

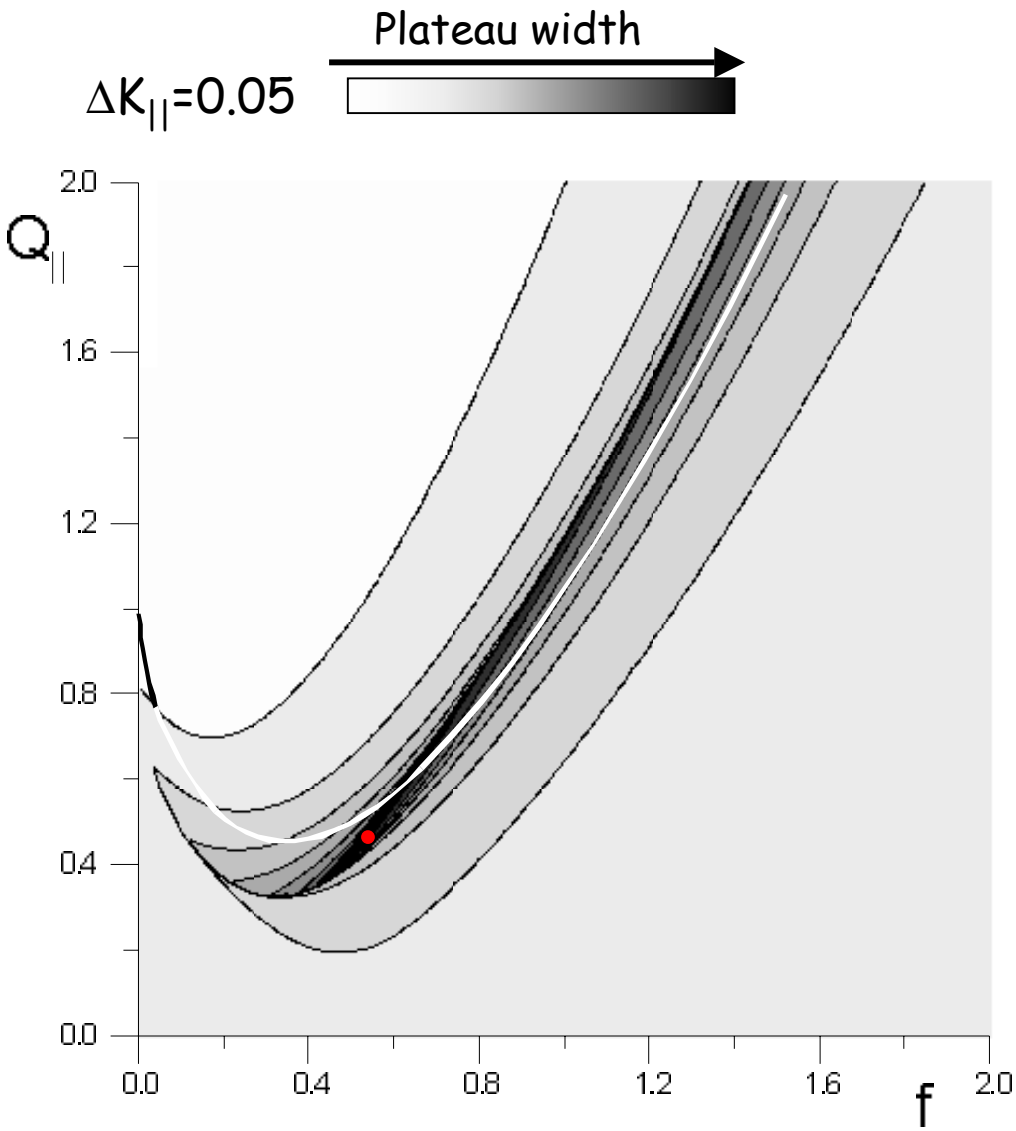



Non-diffractive propagation length

$$L = \pi / \Delta k_{||}$$

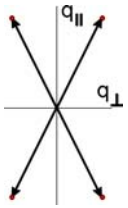
Smallest possible structures

$$\Delta x_{min} = 2 / \Delta k_\perp$$

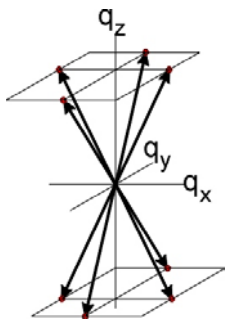


# 3D Nondiffractive Photonic Crystals

2D PC




3D PC



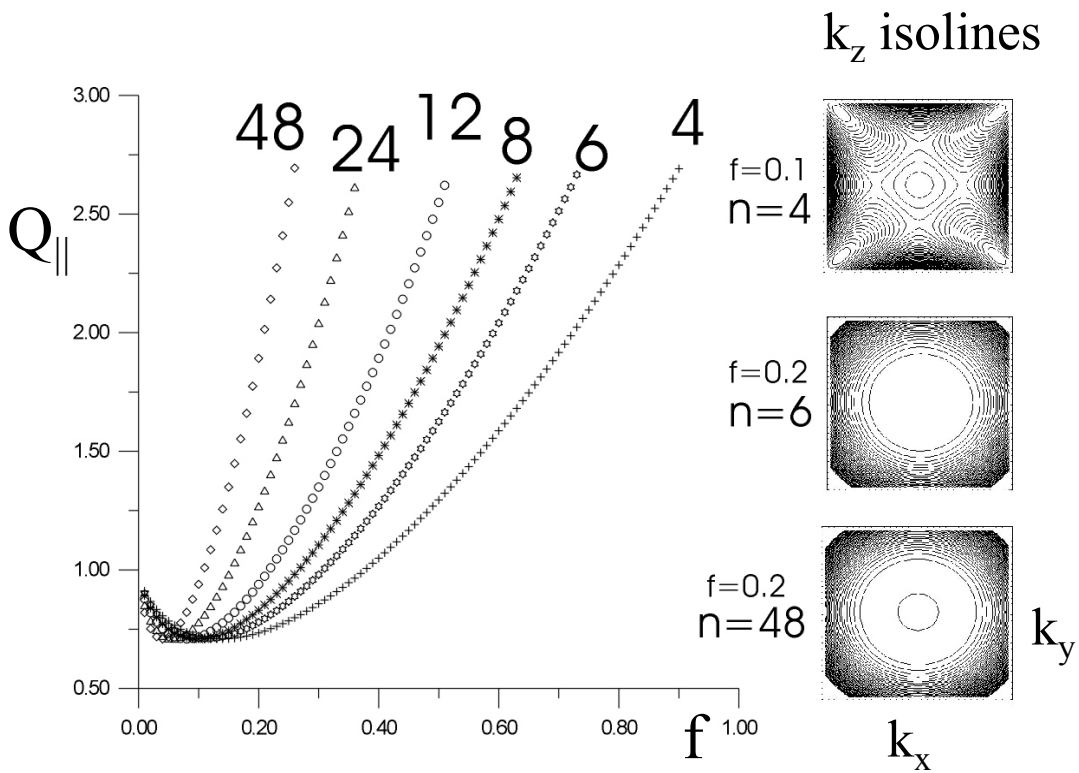

Diferent configurations: n-gons

$$\Delta n(\vec{r}) = 4m\cos(q_{\parallel}z)\sum_{l=0}^{n-1}\cos(\alpha q_{\perp}x + \beta q_{\perp}y + \varphi_l)$$

$$\alpha = \cos(2\pi l / n)$$
$$\beta = \mathrm{sen}(2\pi l / n)$$

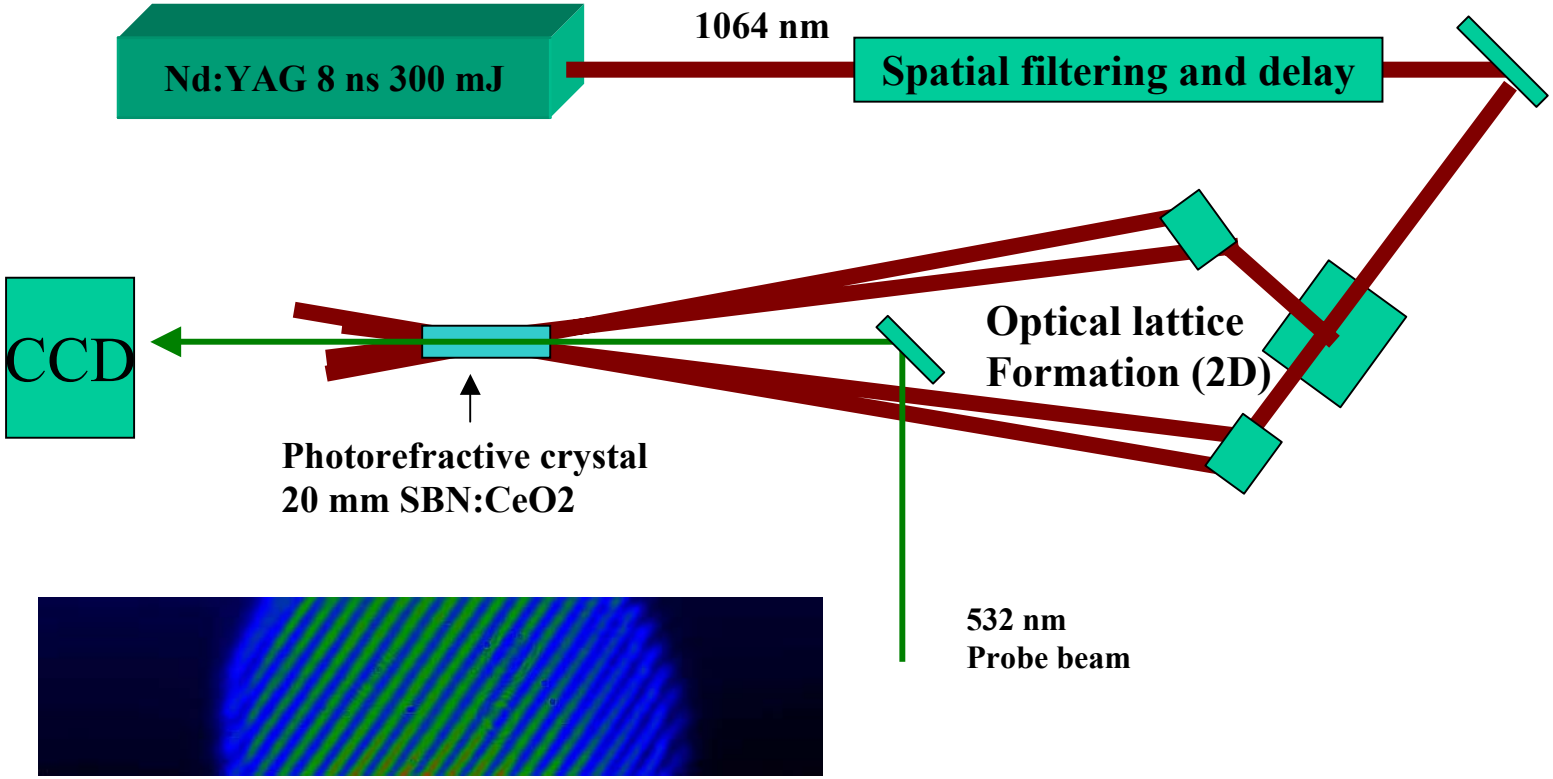
$\varphi_l$ does not effect the dispersion curve → Different PC structures

Quasicrystal, bessel beam

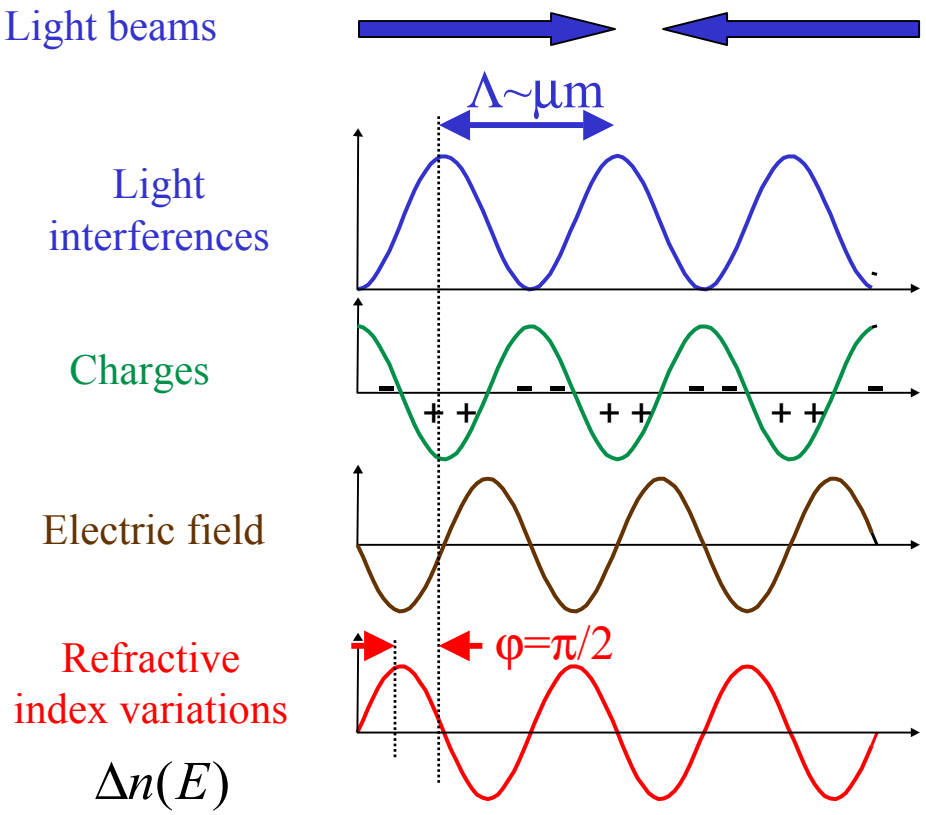



n ↓ More symetric, less distorted propagated patterns ?

# Experiments with PCs



# •Photorefractive materials



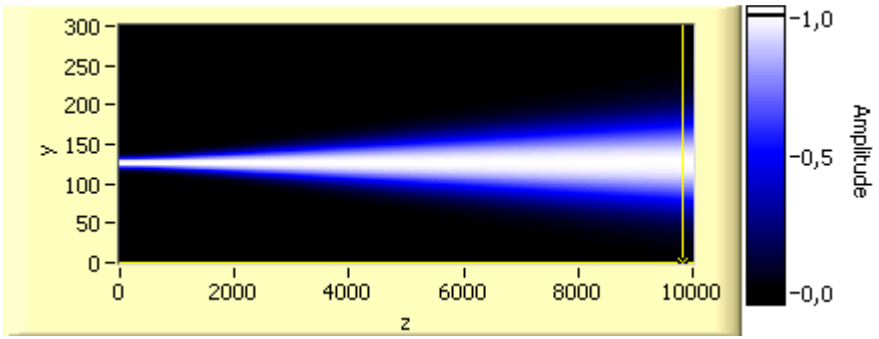
## •Simulations with experimental parameters

•Beam:

Wavelength = 532 nm
Initial width = 12.6 μm

Propagation length = 4.25 mm

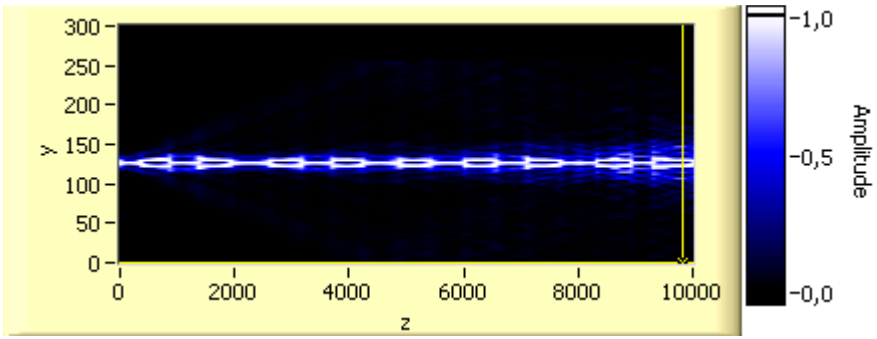
Homogeneous material




•Photonic crystal:

$\Delta n$ modulation amplitude= 5 $10^{-4}$
Spatial periodicity:
$\Lambda_{\perp}$=7.6μm , $\Lambda_{\parallel}$= 483 μm

Photonic crystal



# Possible applications:

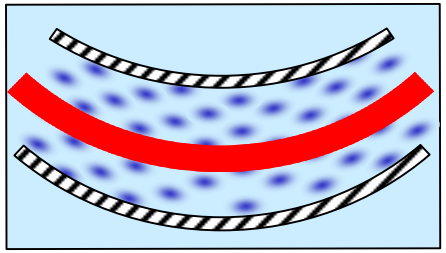
- Multimode nondiffractive wave-guides
  - Fibers transporting patterns
  - Wave-guides in electronic circuits
  - ....

| Diffractive fibers | Nondiffractive fibers |
| --- | --- |
| 1 fiber transport 1 bit (light or no light) | 1 fiber transport 1 pattern |

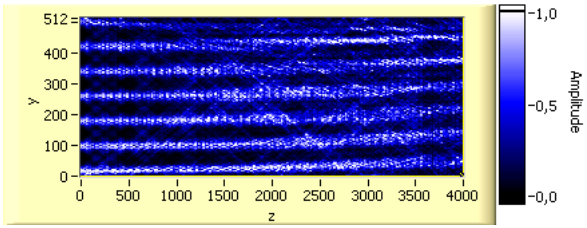
- Microscopy, photolitography, ...

# Multimode Nondiffractive Fibers and Wave-Guides:

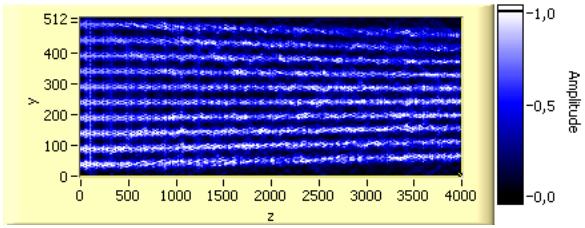
## Bending

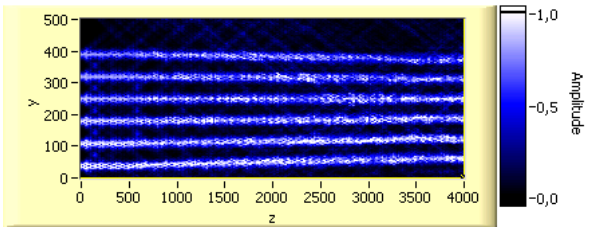


Radi de curvatura = 9 x diàmetre fibra

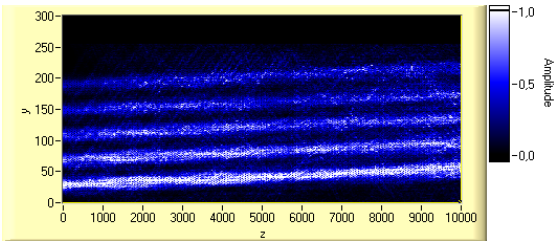



## Lens effect




## Shift in x



# Non-diffractive pulses

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}-\frac{i}{2k_0}\frac{\partial^2}{\partial x^2}-i\Delta n(x,z)k_0\right)A(x,z,t)=0$$

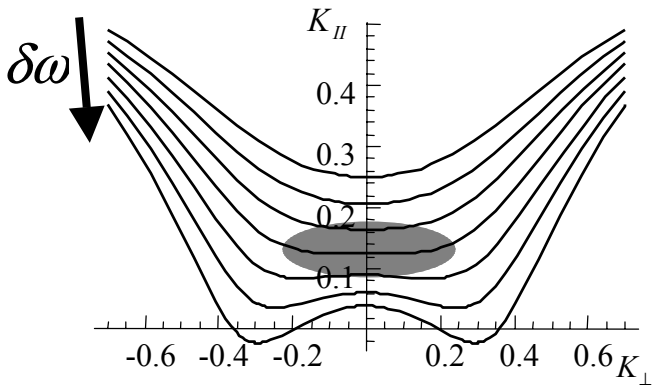
$f = 2mk_0^2/q_\perp^2$ - modulation depth;

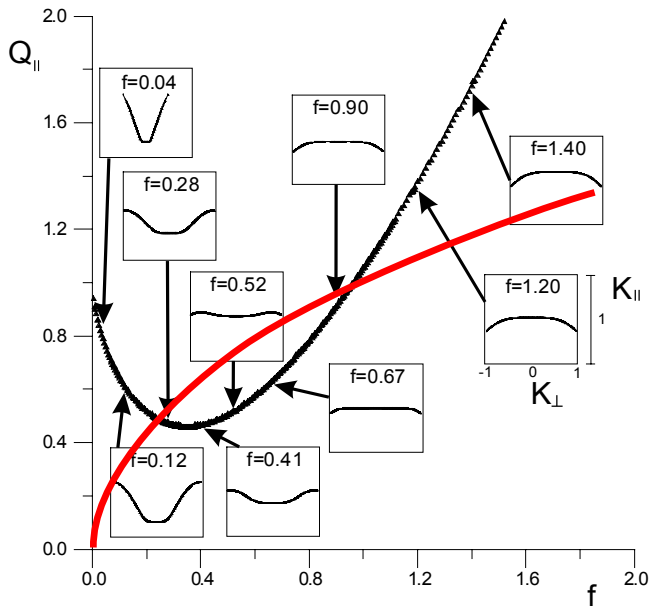
$Q_{II} = 2q_{II}k_0/q_\perp^2$ - geometry;

$$k_0 = \omega_0/c$$

- Carrier frequencies interval

Small variations of the carrier frequency $\omega=\omega_0(1+\delta\omega)$



# Small index modulations (f<<1)

$$K_{II} = K_{II,0} + \frac{\delta\omega}{V_0} + \frac{\delta\omega^2}{4} + \alpha \cdot \delta\omega K_\perp^2$$

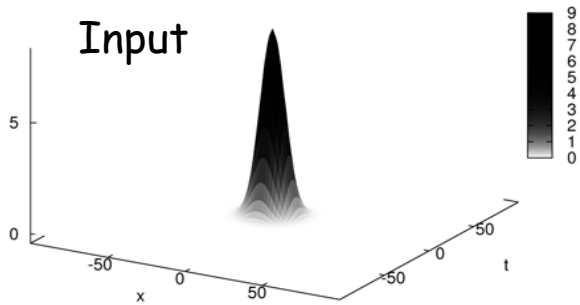
$$K_{II,0} = \frac{\left(1 - Q_{II,0}\right)^2}{4} \qquad \alpha = \frac{3}{\left(1 - Q_{II,0}\right)}$$

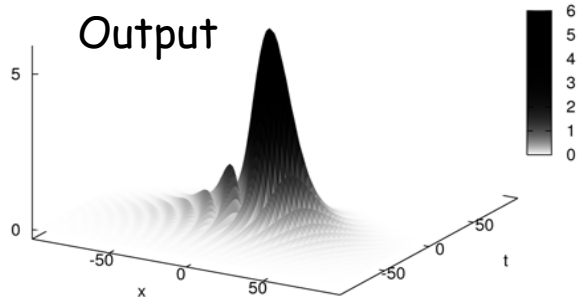
$$\frac{1}{V_0} = \frac{\left(1 - Q_{II,0}\right)\cdot\left(3 - Q_{II,0}\right)}{2}$$

$$\left(\frac{\partial}{\partial Z} - iK_{II,0} + \frac{1}{V_0}\frac{\partial}{\partial T} + \frac{i}{4}\frac{\partial^2}{\partial T^2} - \alpha\frac{\partial}{\partial T}\frac{\partial^2}{\partial X^2}\right)A = 0$$

- Gaussian pulse in x and t

Propagation length: 375 μm



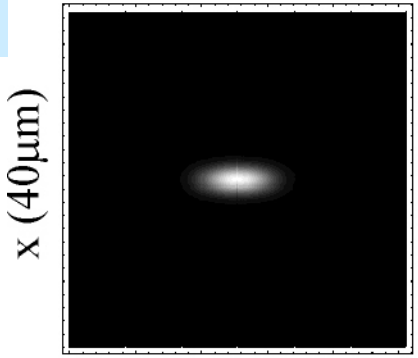
# Integration of the main equation

$$\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}-\frac{i}{2k_0}\frac{\partial^2}{\partial x^2}-i\Delta n(x,z)k_0\right)A(x,z,t)=0$$

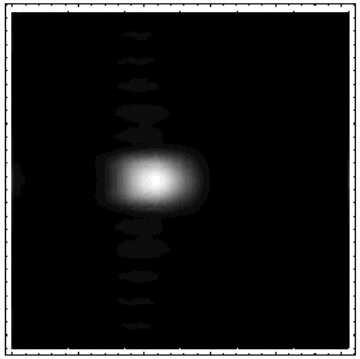
- Gaussian pulse in x and t

**Propagation length: 0.45 mm**

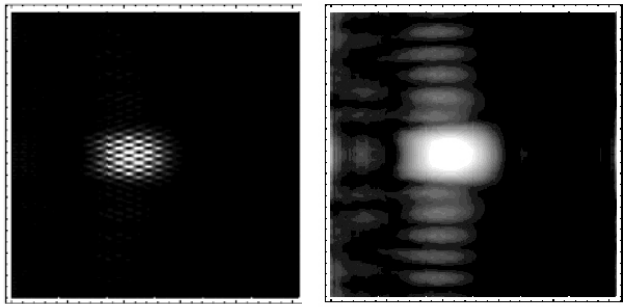
Input

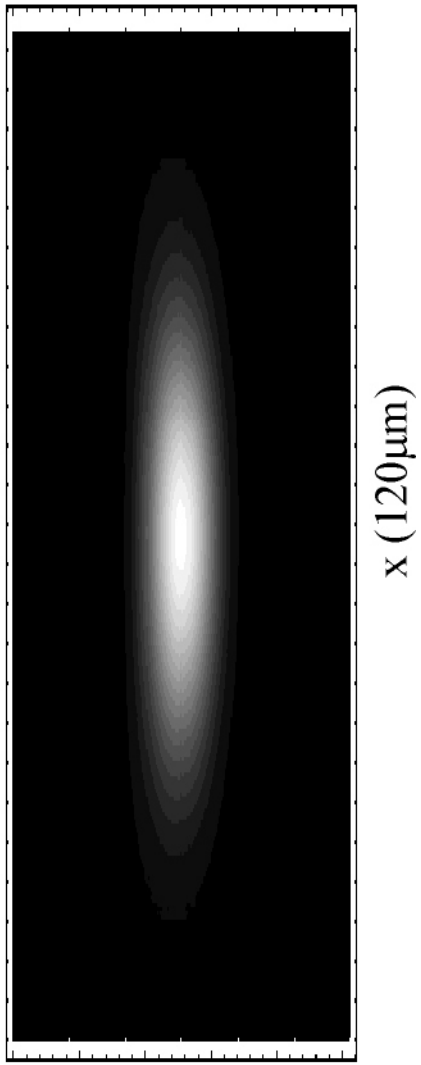



Output with PC



Filtrat per treure les oscil·lacions degudes a la modulació de Δn





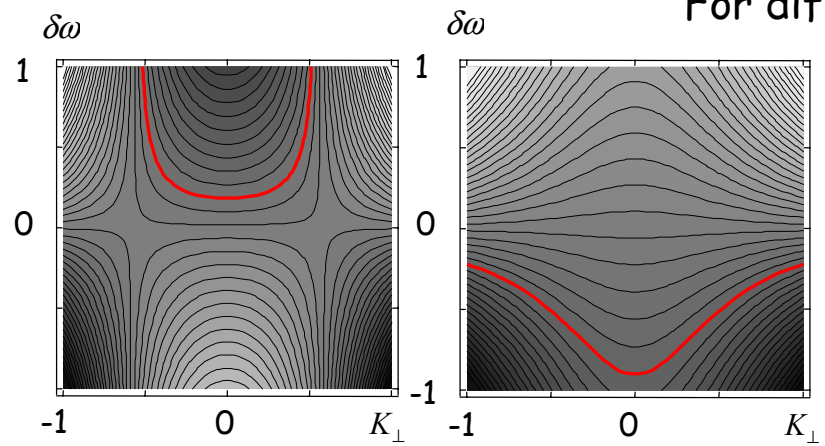

Output without PC

# Invariant spacio-temporal shapes

$$K_{II} = K_{II,0} + \frac{\delta\omega}{V_0} + \frac{\delta\omega^2}{4} + \alpha \cdot \delta\omega K_\perp^2$$

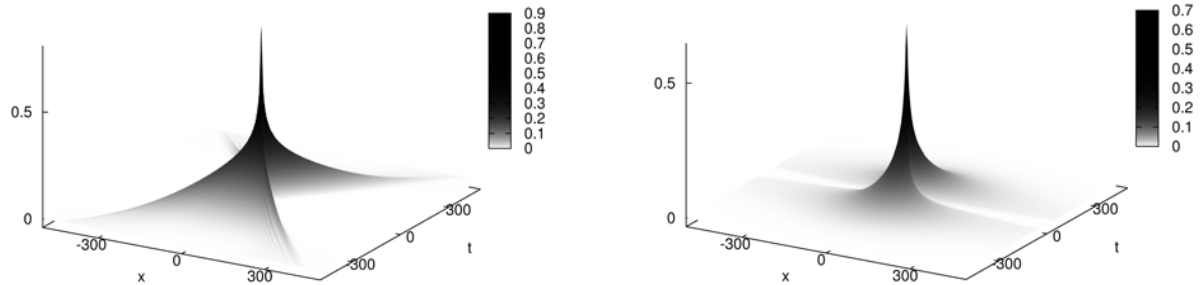
$$K_{II}(K_\perp, \delta\omega) = const$$

For different group velocities $V_0$

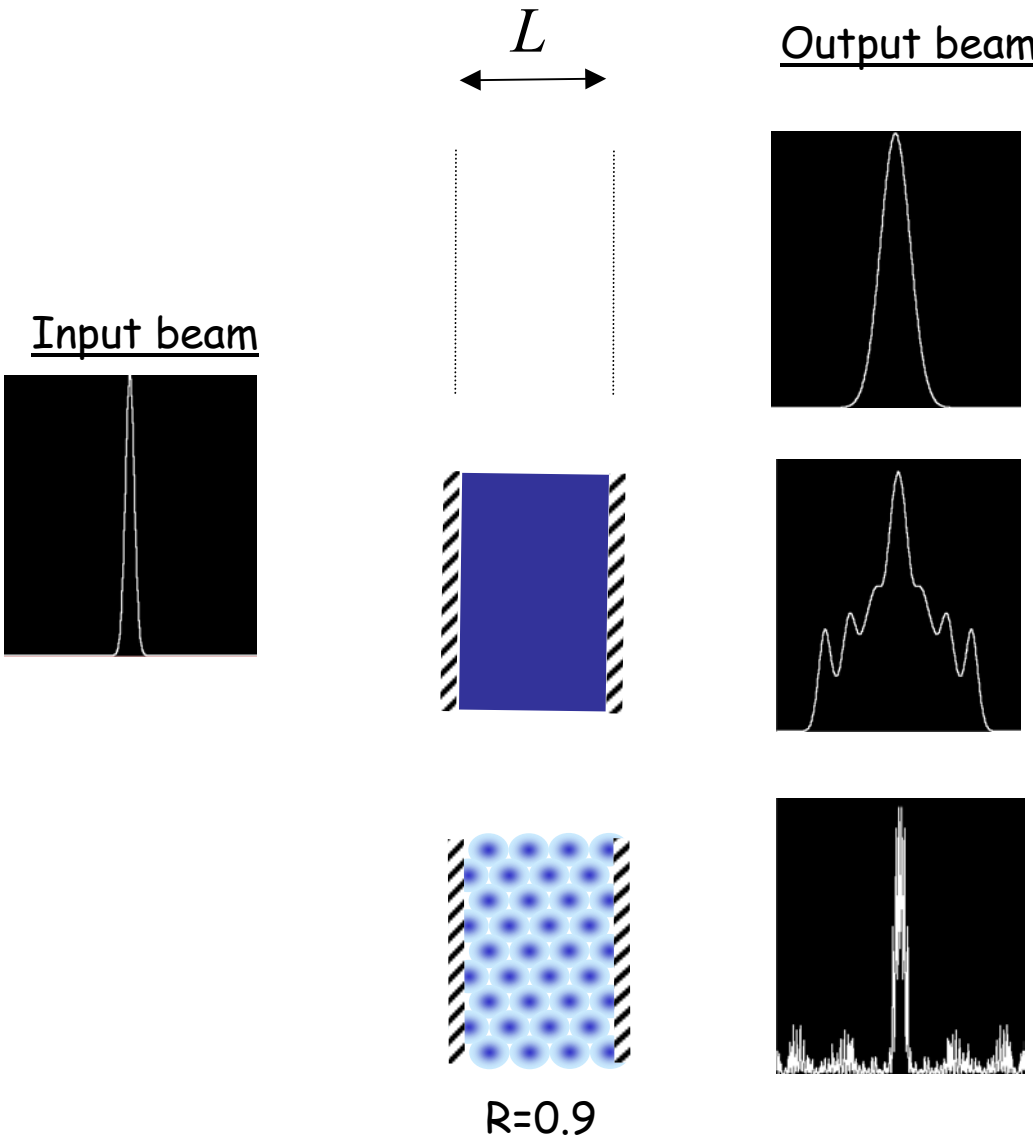



Adding modes of the same isoline

No dispersive, no diffractive Propagation



# Non-diffractive resonators




•Free propagation along distance L

•Homogeneous spacer

Gaussian spectrum filtered by the resonator

Broader beam

•Non-diffractive cavity
Spacer filled with PC

# Homogeneous resonators

Homogeneous spacer

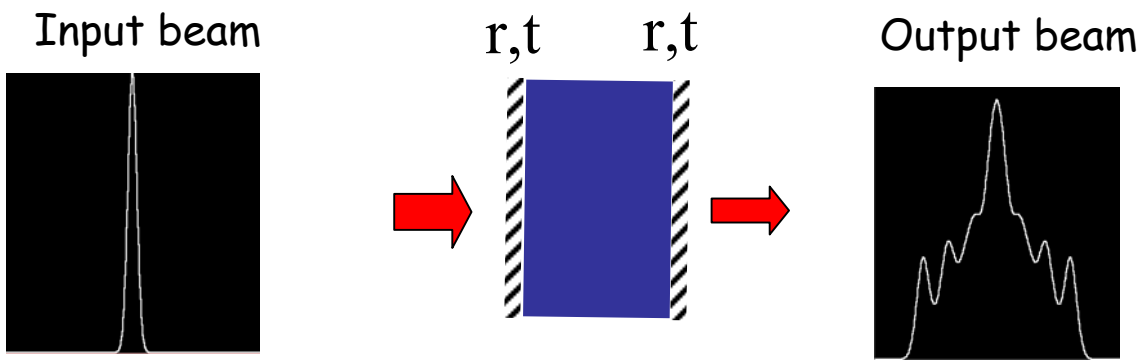
- <u>Transmitted field</u> $A(k_\perp) = A_0(k_\perp)\dfrac{t^2}{1-r^2e^{i\varphi}}$

reflection $r$, transmission $t$ and phase shift φ of the cavity

Different φ for each transverse component $\varphi(k_\perp) \cong \dfrac{-k_\perp^2}{2k_0}2L = -dk_\perp^2$

effective diffraction of the cavity $d=\lambda L/2\pi$.

**<u>Gaussian spectrum filtered by the resonator</u>**

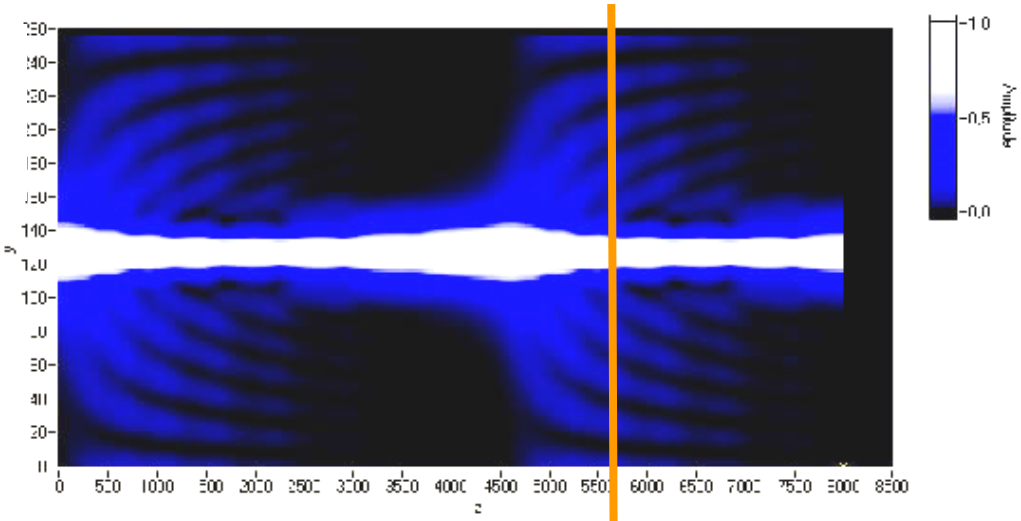


# •Scann of the cavity length

$w_0$
input width

**Output intensity**

Rings



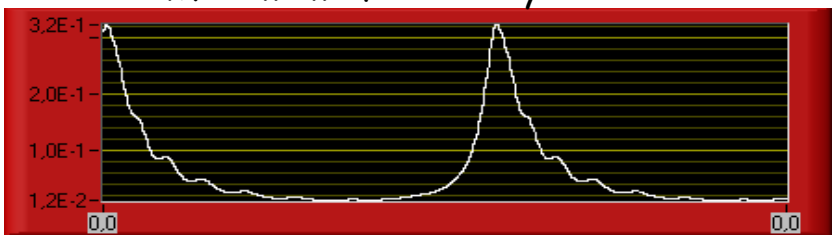
400 λ

400.5 λ

## Diffraction

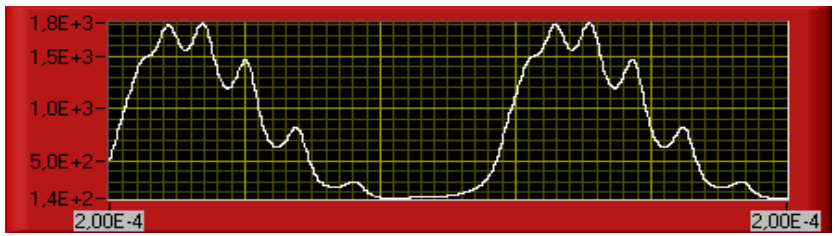
- Rings
- Free spectral range: $\lambda/2+a$ , $a(w_0)$
- Output width depends on L

Maximum of intensity



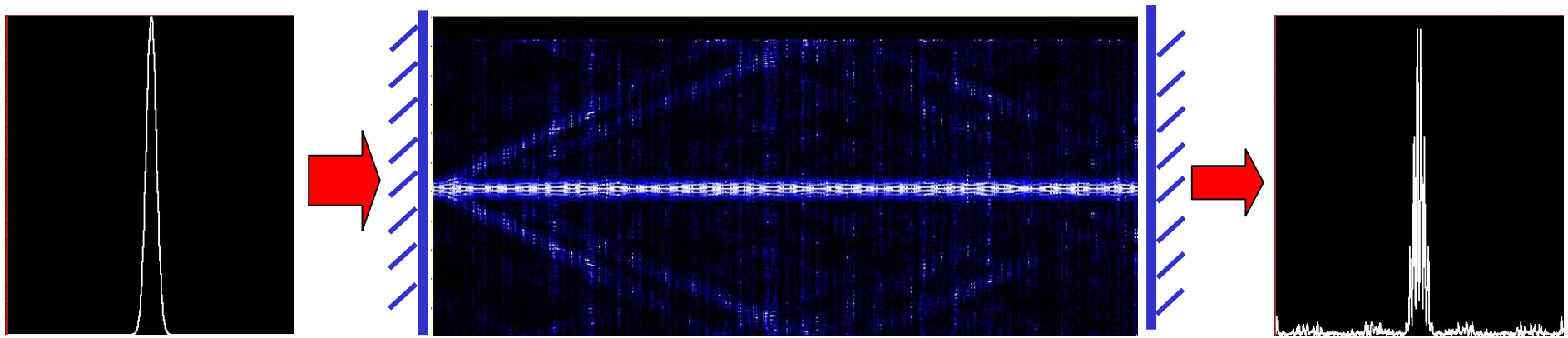

Free spectral range

Waist



# Photonic crystal resonators

## Photonic crystal spacer



•No Rings. Only the central peak remains (all transverse waves in phase).

•Free spectral range does not depend on $w_0$.

•Output width does not depends on L.

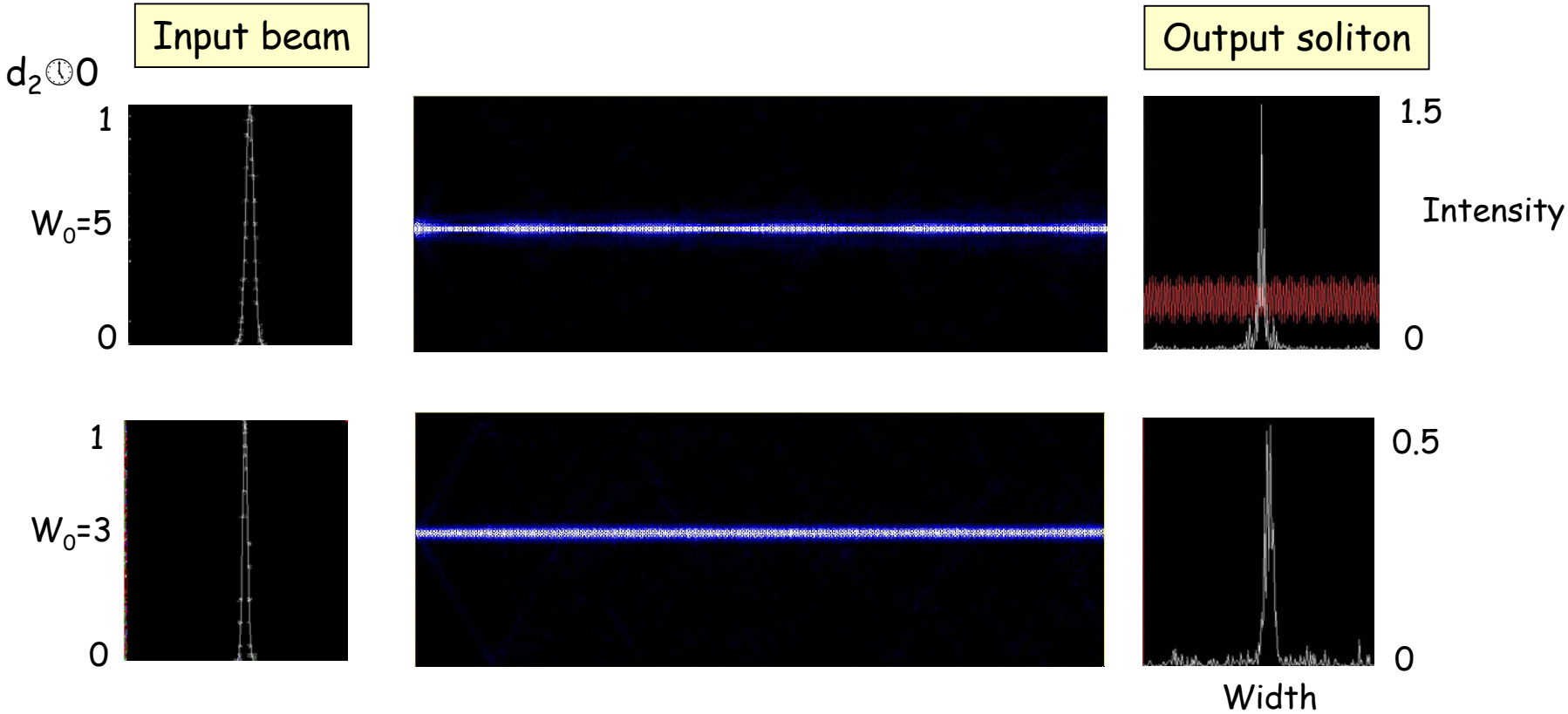
# Propagation in Nonlinear PC's (1D transvers)

$$\left(2ik_0\frac{\partial}{\partial z}+\frac{\partial^2}{\partial x^2}+2\Delta n(x,z)k_0^2-c|A|^2\right)A=0$$

**Diffraction** $\frac{\partial^2 A}{\partial x^2}$ ⇒ ⇐ $c|A|^2$ **Nonlinearity**

⇒ **Solitons**

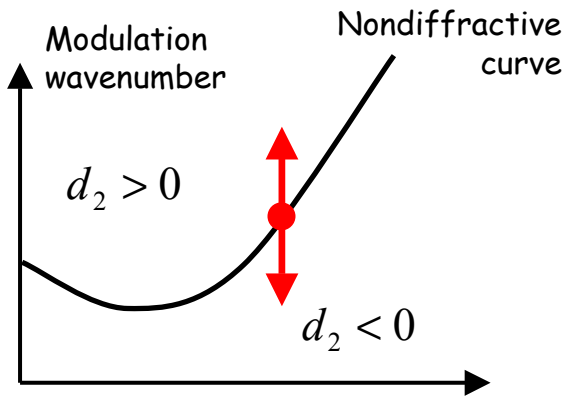
Solitons: Amplitude-Width relationship

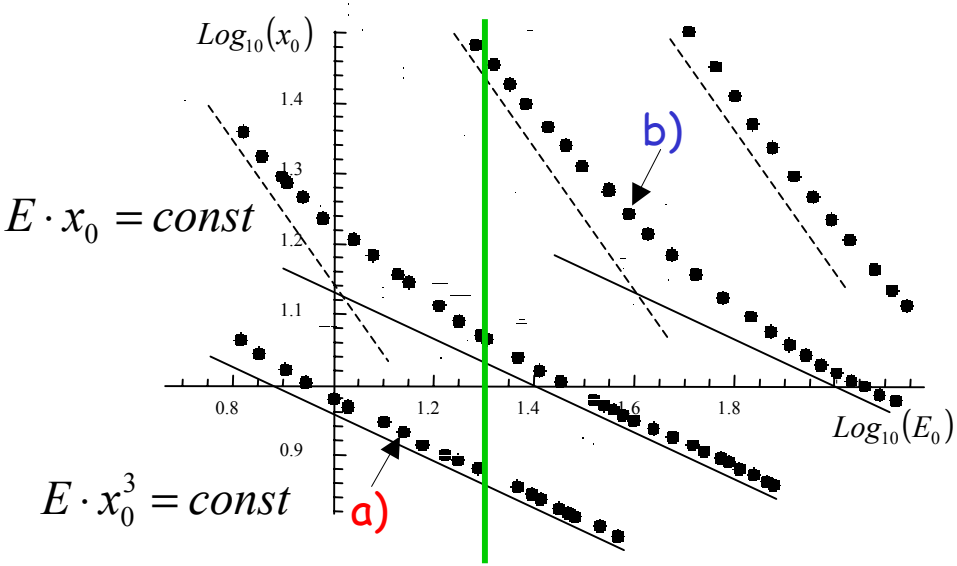


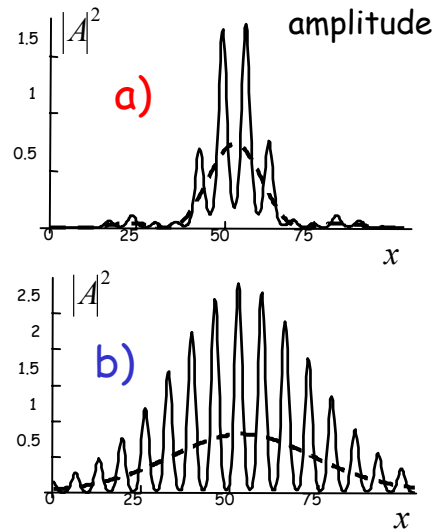
# Near the linear non-diffractive curve

$$\left(2ik_0\frac{\partial}{\partial z}+\frac{\partial^2}{\partial x^2}+2\Delta n(x,z)k_0^2-c|A|^2\right)A=0$$

$$i\frac{\partial A}{\partial T}=\left(d_2\frac{\partial^2}{\partial X^2}-d_4\frac{\partial^4}{\partial X^4}+C_B|A|^2\right)A$$



# Nonlinear PCs (2D transvers)

NLSE: $$\frac{\partial A}{\partial t} = i\left(d_{eff}\nabla^2 - |A|^2\right)A$$

**NLSE colapses in 2D and 3D .**
**No stable solitons appear.**

**Subdiffractive NLSE**

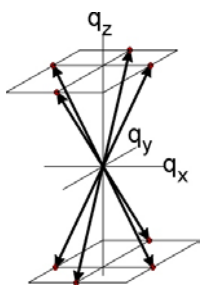
$$\left(2ik_0\frac{\partial}{\partial z} + \nabla^2 + 2\Delta n(x,z)k_0^2 - c|A|^2\right)A = 0$$

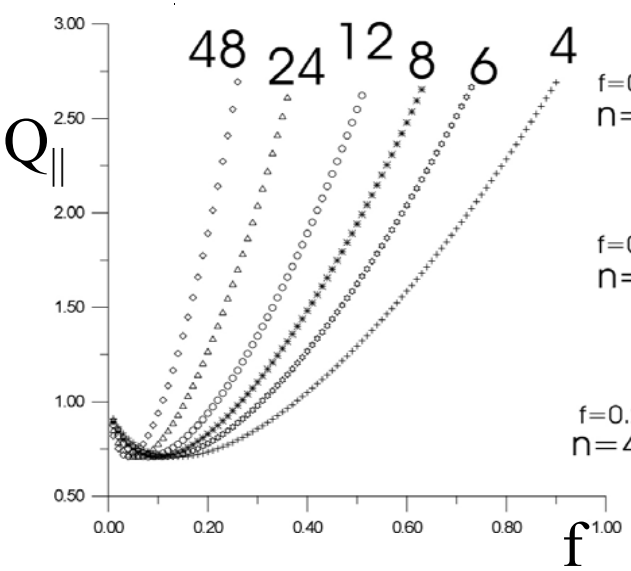
Near $d_2 \to 0$: $$i\frac{\partial A}{\partial Z} = \left(\cancel{d_2\nabla^2} - d_4\nabla^4 + C_B|A|^2\right)A$$

**Stable solitons**
- Analyticaly
- numericaly

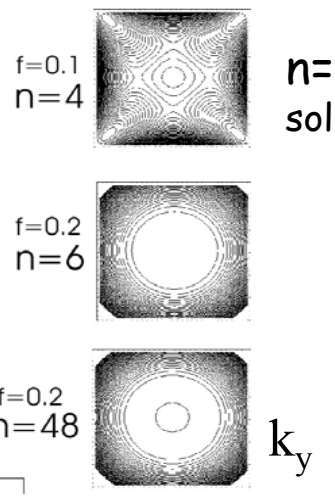
**different geometries:**

Nondiffractive curves

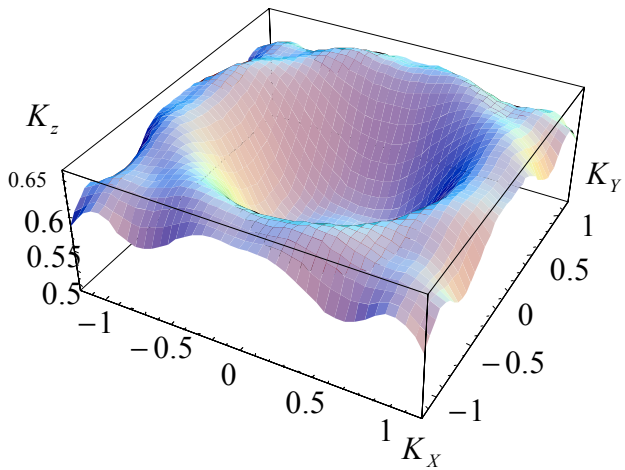



$k_z$ isolines




n=4 not usefull to stabilize nonlinear solitons



# Bose-Einstein Condensates

$$\left(2ik_0\frac{\partial}{\partial z}+\frac{\partial^2}{\partial x^2}+2\Delta n(x,z)k_0^2-c|A|^2\right)A=0$$

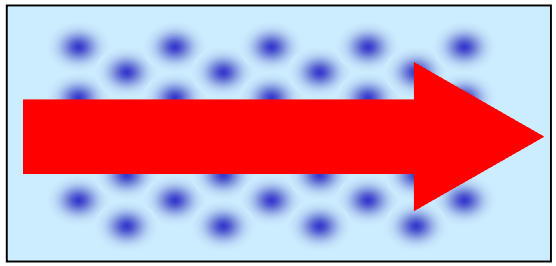
$$i\frac{\partial A}{\partial T}=\left(d_2\nabla^2-d_4\nabla^4+C_B|A|^2\right)A$$

NLSE: $\frac{\partial A}{\partial t}=i\left(d_{eff}\nabla^2-|A|^2\right)A$

- **Optical nonlinearities**
- **Bose-Einstein Cond. (BEC's)**

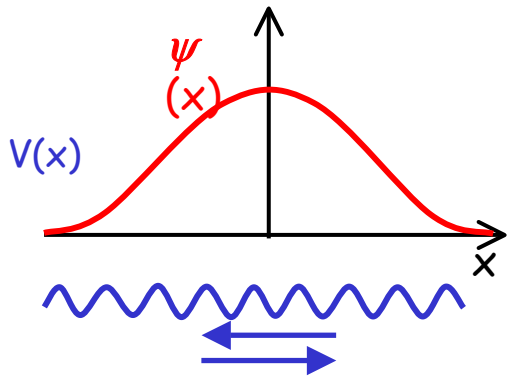
$A \Rightarrow \psi$

## Light beam



Diffraction

## Bose-Einstein Condensate

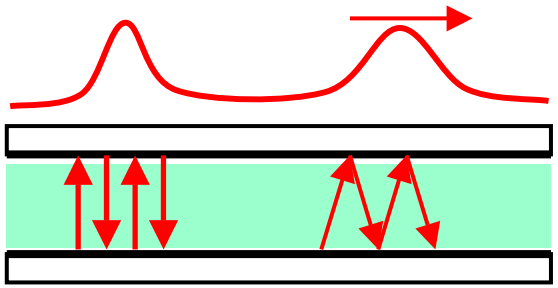



Effective mass

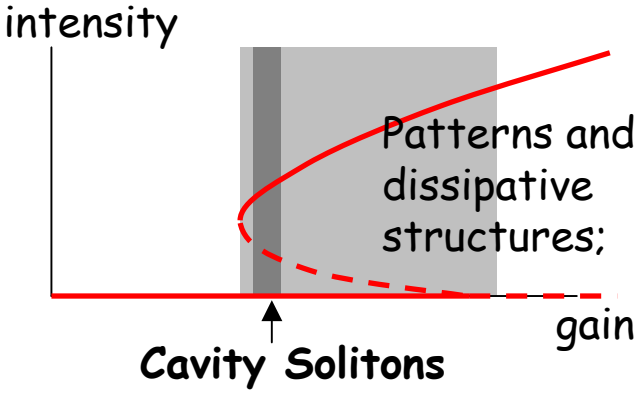
Stable solitons in 2D and 3D

# Non-diffractive Nonlinear resonators

- Nonlinear resonators:



Homogeneous material



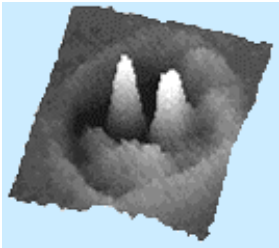

**CV** in VCSELs, lasers with saturable absorber, OPOs, Photorefractives,

$$\partial A(r)/\partial t = \ldots + id\nabla^2 A(r) + \ldots$$

$$size \approx d^{1/2} \approx \sqrt{\lambda \cdot L \cdot Q}$$

$$\lambda \approx 1\,\mu m \quad L \approx 10\,\mu m \quad Q \approx 100$$

$$x_0 \approx 30\,\mu m$$

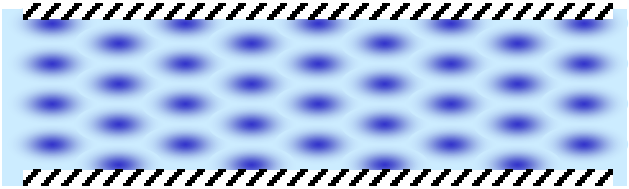


**Switch on/off in any point** → **•Memory •Processing**

Taranenko, Weiss, Kuszelewicz , 2000

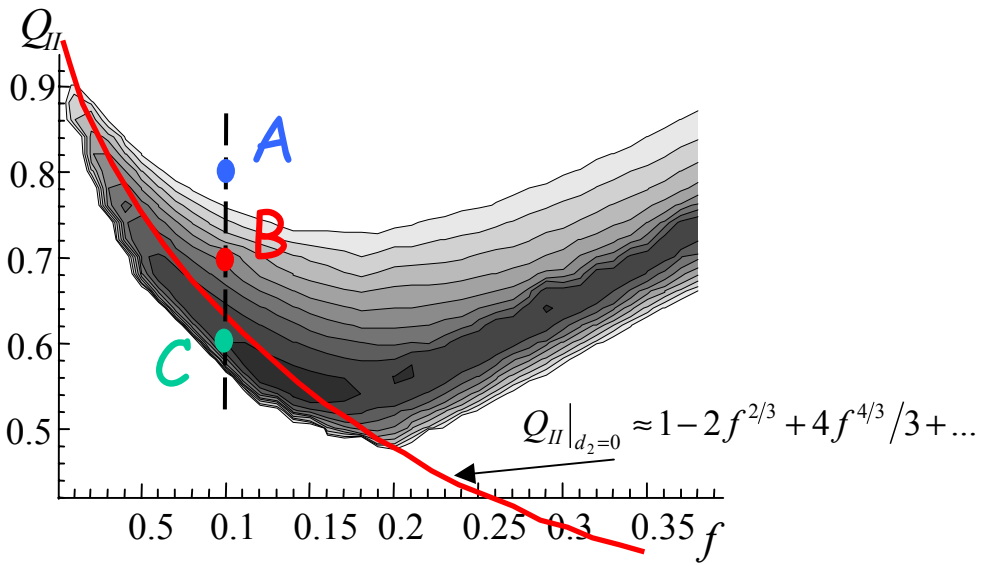
# •Nonlinear photonic crystal resonators:

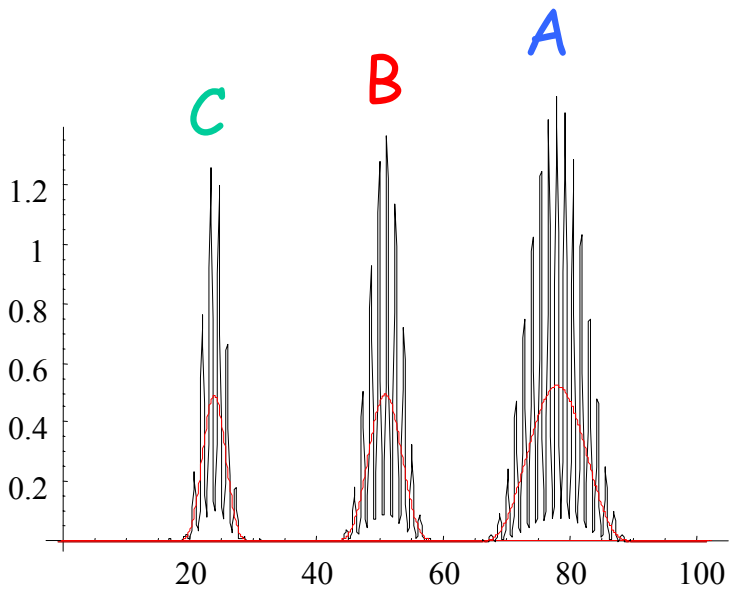
Modulation of refraction index



Reduction of spatial scale

$$size \approx \lambda$$



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•R.Iliew e.a. 2004

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•K. Staliunas, C.Serrat, C.Cojocaru, J.Trull and R. Herrero, *Nonspreading Light Pulses in Photonic Crystals* , subm. 2005, arXiv: physics/0505153, accepted in PRE

•K.Staliunas, R.Herrero, G.Valcarcel and N.Achmediev, *Subdiffractive Solitons in Two-Dimensional Nonlinear Schrödinger Equation* 2006, submitted

•K.Staliunas, G.Valcarcel and R.Herrero, *Sub-Diffractive Band-Edge Solitons in Bose-Einstein Condensates in Periodic Potencials* , 2006, submitted

# More

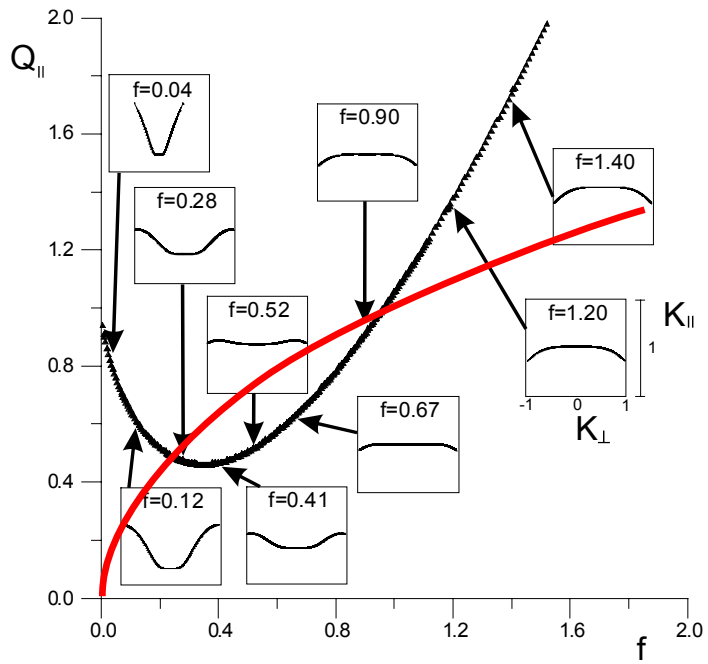
$$f = 2mk_0^2/q_\perp^2$$ - modulation depth;

$$Q_{II} = 2q_{II}k_0/q_\perp^2$$ - geometry;

$$k_0 = \omega_0/c$$

- Sub-sub-diffractive?

  ($Q_{||}$,f) with $\nabla^2 = \nabla^4 = 0$ ➡ $\nabla^6$




- Nondiffractive-nondispersive pulses?